Dynamic Adjustment to Trade Shocks

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Abstract

Global trade flows and supply chains adjust gradually. Empirical estimates of the trade elasticity for the short run are about half as large as those for the long run and suggest that trade is subject to substantive adjustment frictions. We develop a tractable framework that provides microfoundations for dynamic trade adjustment. The model features staggered sourcing decisions, nests the Eaton-Kortum model as the limiting long-run case, and rationalizes reduced-form estimation of horizon-specific trade elasticities. We calibrate the model to horizon-specific trade elasticities and use it to quantify the welfare impact of the 2018 US-China trade war. Staggered sourcing decisions considerably exacerbate losses from the trade war, with cumulative welfare losses 300% larger in the short run and 70% larger in the long run than in the Eaton-Kortum benchmark. Third countries such as Mexico can suffer welfare losses in the short run and welfare gains in the long run.

Keywords: International trade; estimation of the elasticity of trade; dynamic trade adjustment; staggered sourcing decision; US-China trade war

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1 Introduction

Innovations and disruptions to global supply chains lead to gradual adjustments in international trade flows. It has long been recognized that the trade elasticity, a key parameter that captures the substitution between imported goods from different countries in response to trade costs, varies by time horizon (e.g. Dekle, Eaton and Kortum, 2008). Boehm, Levchenko and Pandalai-Nayar (2023) use plausibly exogenous tariff changes to measure the trade elasticity by time horizon and find that the short-run trade elasticity is about half the size of the long-run elasticity. This differential implies substantial frictions in trade adjustment that a static trade model cannot account for. A dynamic framework is needed to provide a rigorous and plausible quantification of the transitory impact of shocks to global supply chains.

This paper proposes a dynamic general-equilibrium model of trade with many countries and many industries, where staggered sourcing decisions give rise to horizon-specific trade elasticities. Under the Ricardian trade tenet, products are sourced from the least expensive global supplier. However, the opportunity to switch to a new supplier only arrives randomly following a Poisson process. As a consequence, only some buyers respond to a trade disruption by adjusting to optimal sourcing relations. Other buyers endure a suboptimal sourcing choice until they can adjust. In this framework, disruptions put the world economy through a sustained period of adjustment.

The model preserves the analytical tractability of a class of quantitative Ricardian models based on Eaton and Kortum (2002, henceforth EK). We characterize impulse responses in the model using the dynamic hat algebra method. We establish a closed-form expression for the horizon-specific trade elasticity, showing that our model rationalizes the estimates in Boehm, Levchenko and Pandalai-Nayar (2023) as a convex combination of fundamental elasticity parameters with related implications for horizon-specific gains from trade.

Specifically, we assume that intermediate goods are produced using constant returns-to-scale technologies and producers differ by productivity drawn from a country-sector specific Fréchet distribution. Trade is subject to iceberg trade costs. An assembler of the final good at a destination d seeks to buy from the least expensive global supplier, but may not be able to constantly switch from one supplier to another. The assembler's sourcing decision is governed by a binary random process: an assembler either chooses the least expensive global supplier of an intermediate good from any source-industry, or the assembler continues purchasing from the same producer as in the preceding period. We can therefore characterize equilibrium as a set of measurable partitions of the space of intermediate goods for each supplier, and then derive the equilibrium distributions. An intermediate good's price at a moment in time equals the initial destination price adjusted for the cumulative changes in iceberg trade costs since the supplier was last elected. We show that a destination country's expenditure shares by source country across intermediate goods take an analytic form as in EK and similar Ricardian frameworks that are consistent with the gravity equation of trade.

The expenditure shares in the augmented gravity equation encode the price that a buyer paid at the time of the last supplier change. Through this unmoved component while a buyer-supplier relationship lasts, cross price effects of substitution are governed by the short-run trade elasticity, similar to an Armington (1969) model. When all supplier-buyer relationships are reset optimally, the gravity expression simplifies to the common gravity equation in an EK framework, so that the long-run trade elasticity prevails. With the equilibrium relationships at hand, we compute impulse responses recursively, and we analytically derive the trade elasticity ε_i^h for each time horizon h after a shock to the global supply network:

$$\varepsilon_i^h \equiv \frac{\partial \log \lambda_{sdi,h}}{\partial \log \tau_{sdi,0}} = -\theta_i \left[1 - (1-\zeta_i)^{h+1}\right] - (\sigma_i - 1)(1-\zeta_i)^{h+1},$$

where $\lambda_{sdi,h}$ is destination country d's expenditure share falling on intermediate goods from source country s in industry i in the hth period after the shock, $\tau_{sdi,0}$ is the trade cost component that is shocked at time 0, θ_i is the long-term trade elasticity as in EK, $\sigma_i - 1$ is the short-term trade elasticity as in Armington, and $\zeta_i \in (0,1)$ is a parameter that describes the frequency at which buyers of intermediate goods from industry i can switch suppliers. The prevailing trade elasticity ε_i^h increases over time in absolute value from the short-run to the long-run level (for the common parametrization $\theta_i > \sigma_i - 1$).

In the long-run, the trade elasticity converges to the familiar Fréchet parameter θ_i as in EK. The rate of convergence depends on the frequency at which buyers can establish a new sourcing relationship ζ_i. The key parameters of our model are therefore identifiable from reduced-form estimates of the trade elasticity at varying time horizons as in Boehm, Levchenko and Pandalai-Nayar (2023). This characterization of the horizon-specific trade elasticity also implies a horizon-specific welfare formula that nests the well-known formula from Arkolakis, Costinot and Rodríguez-Clare (2012) as a special case.

We show how the above results can be used to derive a set of estimation equations for the relevant parameters governing short and long-run trade elasticities, document how existing results from Boehm, Levchenko and Pandalai-Nayar (2023) can be employed, and quantify our trade model for 12 aggregate industries and 20 countries. We apply the model to the episode of the US-China trade war in 2018 and show that rich sectoral dynamics can result, with consequential changes in welfare implications. For instance, accounting for the dynamic costs of supply disruptions raises the welfare costs of the trade war in the U.S. by about 70%, compared to a long-run model. Further, gains from trade can qualitatively differ between the short-run and long-run. In the short-run, the price disruptions caused by the US-

China trade war propagate through the network of existing supply relationships, leading to a global reduction in economic welfare. Those short-run losses, in part, reflect the limited scope for third-party countries to gain from the trade dispute by forming new supply relationships with the US or China; however, such gains may materialize in the long-run. As a consequence, countries whose previous trade linkages leave them most exposed to the US-China trade war, such as Mexico, experience large initial welfare losses in the short-run, but sizeable increases in welfare in the long-run.

The wide discrepancy between a low (short-run) trade elasticity in international macroeconomics and a high (long-run) trade elasticity in international trade has been documented in, for example, Ruhl (2008, who calls the discrepancy an "international elasticity puzzle") and Fontagné, Martin and Orefice (2018). Fontagné, Guimbard and Orefice (2022), Boehm, Levchenko and Pandalai-Nayar (2023) and Anderson and Yotov (2022) offer estimation procedures to separately identify short- and long-run trade elasticities. Anderson and Yotov (2022) rationalize their estimation procedure with firm heterogeneity in lag times from recognition to action in the spirit of Lucas and Prescott (1971). In an alternative approach from a macroeconomic perspective, Yilmazkuday (2019) proposes a framework with nested CES models and derives the trade elasticity as the weighted average of macro elasticities. Our general equilibrium model offers a rationalization for the existing estimation methods with a mixture of the Armington and EK elasticities.

The importance of staggered contracts for trade and exchange rate dynamics has been recognized since at least Kollintzas and Zhou (1992) and shares features with staggered pricing (Calvo, 1983). We generalize deterministic contract ages to supplier relationships that end stochastically and to be reset optimally. In a related approach, Arkolakis, Eaton and Kortum (2011) embed a consumer with no knowledge of the identity of source countries into an EK model. The consumer can switch to the lowest-cost supplier at random intervals but cannot act strategically because the supplier is unknown. We rationalize consumer behavior by introducing an assembler that operates similar to a wholesale or retail firm in that it sources bundles of goods at lowest cost while the consumer cannot unbundle the assembled final good. An assembler, in turn, cannot incur losses in imperfect capital markets and thus sources from the current lowest-cost supplier. Our model allows us to derive a stationary equilibrium distribution of supplier prices by age of contract beyond a binary characterization in Arkolakis, Eaton and Kortum (2011).[1] Based on the mixture of the stationary equilibrium distributions of prices by contract age, we can fully characterize steady states as well as transitionary dynamics. As a result, we obtain the original EK model as the limit of the equilibria along the transition path. Our welfare formula therefore endogenously inherits the long-run elasticity as a special case when all supplier contracts are optimally set.

[1] The underlying stochastic process shares features with the so-called Sisyphos Process (Montero and Villarroel, 2016).

The remainder of the paper is organized as follows. We present the model in Section 2, with details on mathematical derivations relegated to the Appendix. In Section 3 we turn to the dynamic analysis of the model. Estimation of the key parameters follows in Section 4. To illuminate the novel dynamic features of the model for the allocation of economic activities during the adjustment path and the welfare consequences, we present a case study of the US-China trade war in Section 5. Section 6 offers concluding remarks.

2 Model

2.1 Fundamentals

Consider a world economy with N destination countries $d \in \mathcal{D} := \{1, 2, \cdots, N\}$, $s \in \mathcal{D}$ source countries of trade flows, and I industries $i, j \in \mathcal{I} := \{0, 1, 2, \cdots, I\}$. Time t is discrete. Subscripts sdi, t denote a trade flow from source region s to destination d in industry i at time t. Households inelastically supply a single production factor (labor) to domestic firms, and markets are perfectly competitive.

Households. In each period t, a mass of L_d infinitely-lived households in country d inelastically supplies one unit of the production factor to domestic firms at a competitive wage $w_{d,t}$. Household utility in country d at time t is given by $u(C_{d,t})$, where $C_{d,t}$ is the final good: a Cobb-Douglas aggregate over the composite goods $C_{di,t}$ from each industry with

$$C_{d,t} = \prod_{i \in \mathcal{I}} \left(C_{di,t}\right)^{\eta_{di}}. \tag{1}$$

The coefficient η_{di} is the consumption expenditure share of industry $i's$ composite good, with $\sum_{i \in \mathcal{I}} \eta_{di} = 1$. Let $P_{di,t}$ denote the price index of the industry i good in d at time t. Country $d's$ consumer price index is then given by $P_{d,t} = \prod_{i \in \mathcal{I}} \left(P_{di,t}/\eta_{d,i}\right)^{\eta_{di}}$. We assume that households consume their income in every period and discount future utility flows at rate $\beta \in (0, 1)$.

Intermediate Goods. Every industry i consists of a continuum of producers of intermediate goods $\omega \in [0, 1]$. For each intermediate good, there is a large set of potential producers in each country with different technologies to produce the good. In each industry, producers of an intermediate good ω have an individual productivity z and operate a constant-returns-to-scale technology to produce the good using domestic labor ℓ and composite goods M_{ji} sourced from other industries:

$$y_i(\omega) = z\,(\ell)^{\alpha_{di}} \prod_{j \in \mathcal{I}} (M_{ji})^{\alpha_{dji}}. \tag{2}$$

where $y_i(\omega)$ is the output of good ω. The coefficient α_{di} is the value-added share of industry i and the parameters $\alpha_{dij} \geq 0$ are such that $\alpha_{di} = \sum_{j \in \mathcal{J}} \alpha_{dji}$.

We assume that intermediate goods can be traded across countries subject to an iceberg transportation cost, which implies that shipping one unit of a good in industry i from country s to country d at time t requires producing $d_{sdi,t} \geq 1$ units in s, where $d_{ddi,t} = 1$ for all d. Moreover, goods imported by d from s at t may be subject to an ad-valorem tariff $\bar{\tau}_{sdi,t}$. We combine both trade costs into one parameter $\tau_{sdi,t} \equiv d_{sdi,t}\bar{\tau}_{sdi,t}$.

Given this formulation of trade costs and technologies, there is a *common unit cost component* at destination d for all intermediate goods produced in country s, which we denote with

$$c_{sdi,t} \equiv \Theta_{sj}\tau_{sdi,t}\,(w_{s,t})^{\alpha_{si}} \prod_{j \in \mathcal{J}} (P_{sj,t})^{\alpha_{sji}}, \tag{3}$$

where Θ_{sj} is a collection of Cobb-Douglas coefficients. The resulting unit cost of good ω at destination d produced in country s with a productivity $z(\omega)$ is given by $c_{sdi,t}/z(\omega)$.

Production technologies for intermediate goods arrive stochastically and independently at a rate that varies by country and industry. In particular, we follow Eaton and Kortum (2012) in assuming that the mass of intermediate goods ω in country s's industry i that can be produced with a productivity higher than z to be distributed Poisson with mean $A_{si}z^{-\theta_i}$.

Assembly of Composite Goods. In each industry, assemblers bundle intermediate goods into a composite good for consumption or production. An assembler procures intermediate goods at the lowest possible price and costlessly aggregates the sourced intermediates into $Y_{di,t}$ units of industry i's composite good using the technology

$$Y_{di,t} = \left(\int_{[0,1]} y_{di,t}(\omega)^{(\sigma_i - 1)/\sigma_i} \mathrm{d}\omega \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{4}$$

where $y_{di,t}(\omega)$ is the quantity purchased of an intermediate good ω by an assembler in country d, and σ_i is the elasticity of substitution between intermediate goods in industry i. We let $p_{di,t}(\omega)$ denote the lowest possible price at which an intermediate good ω can be purchased at destination d. We will explain the exact price at which this intermediate good is available in greater detail below. As we elaborate in Appendix B.1, cost minimization given (4) implies that the price of industry i's composite good at destination d satisfies

$$P_{di,t} = \left(\int_{[0,1]} p_{di,t}(d\omega)^{-(\sigma_i - 1)} \mathrm{d}\omega \right)^{-\frac{1}{\sigma_i - 1}}. \tag{5}$$

2.2 Sourcing Decisions and Trade Flows

Under the Ricardian trade tenet, assemblers seek to source an intermediate good from the least expensive global supplier. However, an assembler may not have the opportunity to adjust its choice of suppliers at any given time due to a sourcing friction, which we describe now. For every intermediate good ω, there is a continuum of producers in every country. Under perfect competition, an assembler optimally sources any given intermediate good ω from only one source country when given the choice.

The assemblers' choice of source country for any given intermediate good ω is governed by an i.i.d. random variable $x_{i,t}(\omega) \in \{0, 1\}$ for each industry. If $x_{i,t}(\omega) = 1$, that is if the global draw for an intermediate good ω from industry i gives all assemblers worldwide the green light to switch to their preferred source country, then all assemblers optimally choose to purchase from the least costly source country for variety ω in industry i at time t. Between assemblers in different countries the optimal source country can vary because of different trade costs. Else, if $x_{i,t}(\omega) = 0$, that is if the global draw for intermediate ω turns to red for all assemblers worldwide, then all assemblers must purchase their intermediate goods ω in industry i from the same producer as in the preceding period $t - 1$. While the identity of the source country does not change, the quantity procured and the price that the assembler pays can differ from the preceding period if the factory gate price moves (because of changing factor costs) or the currently prevailing trade cost moves.

This formulation of sourcing frictions captures search costs and other types of impediments that prevent the optimal rematch of supply relationships at a moment in time. An implication of the sourcing friction is that price elasticities of demand will differ across intermediate goods according to when their suppliers were last chosen. Let $\Omega^k_{j,t}$ denote the set of industry j goods whose supplier at time t was last chosen k periods ago:

$$\Omega^k_{i,t} = \left\{ \omega : x_{di,t-k}(\omega) = 1, \ \prod_{\varsigma=t-k+1}^{t} x_{di,\varsigma}(\omega) = 0 \right\}, \tag{6}$$

where $\cup_k \Omega^k_{j,t} = [0, 1]$. The sets $\Omega^k_{i,t}$ mutually exclusively and exhaustively partition the unit interval of intermediate goods for each industry i.

2.2.1 Demand for Intermediate Goods with Newly Formed Supply Relationships

We now describe the global demand for intermediate goods in each of these sets, beginning with those that are concurrently formed, $\omega \in \Omega^0_{dj,t}$.

If country s is chosen by an assembler in destination d to supply industry i's intermediate good ω at time t, the combination of the producer's productivity ω, factor cost in source country s and the trade cost between s and d in industry i must make the intermediate good the least expensive.

Let $z_{si}(\omega)$ denote the highest realized productivity by any producer in country-industry si. Similar to Eaton and Kortum (2002), our distributional assumptions imply that z_{si} has a country-industry specific Fréchet distribution given by[2]

$$\Pr\left[z_{si}(\omega) \leq z | A_{si}, \theta_i\right] = \exp\left\{-A_{si} z^{-\theta_i}\right\}. \tag{7}$$

For an assembler in destination d the price of an intermediate good ω from the cheapest available source country at time t is

$$p_{di,t}(\omega) = \min_{s\in\mathcal{D}} \left\{\frac{c_{sdi,t}}{z_{si}(\omega)}\right\} \tag{8}$$

for the common unit cost component $c_{sdi,t}$ given by (3) and the producer with the highest realized productivity $z_{si}(\omega)$ in country-industry si.

As in Eaton and Kortum (2002), the distribution of paid prices across intermediate goods in the set $\Omega^0_{i,t}$ in destination d at time t satisfies

$$G^0_{di,t}\left[p_{di,t}(\omega) \leq p\right] \equiv \Pr\left[p_{di,t}(\omega) \leq p \middle| x_{i,t}(\omega) = 1\right] = 1 - \exp\left\{-\Phi^0_{di,t} p^{-\theta_i}\right\}, \tag{9}$$

where

$$\Phi^0_{di,t} \equiv \sum_{n\in\mathcal{N}} A_{ni} [c_{ndi,t}]^{-\theta_i} \tag{10}$$

is a measure of destination d's market access for intermediate goods $\omega \in \Omega^0_{i,t}$, given trade cost and factor prices behind the common unit cost component $c_{ndi,t}$ by (3). We relegate the derivation of these results to Appendix B.2. To guarantee that the distribution of paid prices has a finite mean later, we impose the standard parametric restriction that $\theta_i > \sigma_i - 1$ for all $i \in \mathcal{I}$.

The properties of the Fréchet distribution imply that $G^0_{di,t}$ also equals the distribution of prices for

[2] Our model could also accommodate productivity change over time with a country-industry-time specific Fréchet distribution and resulting $z_{si,t}(\omega)$ realizations that vary over time. To focus most sharply on adjustment to trade shocks, we do not specify productivity shocks.

intermediate goods $\omega \in \Omega_{i,t}^{0}$ sourced from any source country s. As a result, country d's expenditure share for each potential source country s across intermediate goods $\omega \in \Omega_{i,t}^{0}$ must equal the probability that this source country offers the lowest global price:

$$\lambda_{sdi,t}^{0} = \frac{A_{sj}[c_{sdi,t}]^{-\theta^{i}}}{\Phi_{di,t}^{0}}. \tag{11}$$

with the common unit cost component $c_{sdi,t}$ given by (3).

Within the set of intermediate goods that are sourced through concurrently and optimally formed supply relationships, the partial equilibrium elasticity of trade flows with respect to trade cost is governed by the familiar Fréchet parameter:

$$\left.\frac{\partial \log \lambda_{sdi,t}^{0}}{\partial \log \tau_{sdi,t}}\right|_{\Phi_{di,t}^{0}} = -\theta_{j}.$$

2.2.2 Demand for Intermediate Goods with Continuing Supply Relationships

Intermediate goods $\omega \in \Omega_{j,t}^{k}$ are purchased from a supplier that was chosen at time $t-k$. To characterize prices and expenditure allocations across these intermediate goods at time t, we denote changes over time for a variable x_t succinctly by $\hat{x}_t \equiv x_t/x_{t-1}$.

Suppose an assembler in d first sourced an intermediate good ω from s at time $t-k$ under the unit input cost $c_{sdi,t-k}/z_{si}(\omega)$, which depends on equilibrium factor prices and parameters by the common unit cost component (3). If the intermediate good is still sourced from the same producer at time t, its price will then equal:[3]

$$p_{sdj,t}(\omega) = \frac{c_{sdi,t}}{z_{si}(\omega)} = \frac{c_{sdi,t-k}\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sid,\varsigma}}{z_{si}(\omega)}, \tag{12}$$

which is the initial destination price adjusted for the cumulative changes in iceberg trade costs and factor cost since $t-k$.

We show in Appendix B.3 that country d's expenditure share by source country across intermediate goods $\omega \in \Omega_{i,t}^{k}$ equals

$$\lambda_{sdi,t}^{k} = \frac{\lambda_{sdi,t-k}^{0}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sid,\varsigma}\right)^{1-\sigma_i}}{\Phi_{di,t}^{k}}, \tag{13}$$

[3]Note that $x_t = x_{t-k}\frac{x_{t-k+1}}{x_{t-k}}\cdots\frac{x_t}{x_{t-1}} \equiv x_{t-k}\hat{x}_{t-k+1}\cdots\hat{x}_t$. For a composite variable such as $c_{sdi,t} = \tau_{sdi,t}\,w_{s,t}$, the change over time is $\hat{c}_{sdi,t} = \hat{\tau}_{sdi,t}\,\hat{w}_{s,t}$.

where

$$\Phi_{di,t}^{k} \equiv \sum_{n\in\mathcal{N}} \lambda_{ndi,t-k}^{0} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{nid,\varsigma} \right)^{1-\sigma_i} \tag{14}$$

reflects the mean price that a buyer pays for the set of intermediate goods $\Omega_{i,t}^{k}$ at time $t-k$ through the trade shares $\left\{ \lambda_{nid,t-k}^{0} \right\}_{n\in\mathcal{N}}$.

Comparing Equations (11) and (13) shows how cross-price effects differ across intermediate goods depending on when a supply relationship is formed. If assemblers can source from the least expensive global supplier of an intermediate good at time t, cross-price demand effects are governed the Fréchet parameter θ_i, and trade is governed by comparative advantage.

Conversely, if an assembler is unable to switch suppliers, then the extensive margin is shut down. The only margin of adjustment is the intensive margin, which is captured by the terms that collect the product of changes in unit input costs. Effectively, over those partitions, trade happens as if varieties were differentiated across countries with the measure of varieties of each source defined at the last period of adjustment —i.e. at period $t-k$ for partition $\Omega_{i,t}^{k}$.

In order words, for each partition $\Omega_{i,t}^{k}$, trade happens under Armington forces. Intuitively, the price elasticity of demand is governed by the elasticity of substitution $\sigma_i - 1$, which captures Armington trade:

$$\left. \frac{\partial \log \lambda_{sdi,t}^{k}}{\partial \log \tau_{sdi,\varsigma}} \right|_{\Phi_{di,t}^{k}} = -\left(\sigma_i - 1\right) \quad \text{for } t-k < \varsigma < t.$$

To close the model, we now show how aggregate global demand for industry i's composite good follows from aggregating the trade shares in Equations (11) and (13).

2.3 Aggregation

To find aggregate demand, we leverage the homotheticity of assembly. The partial price index for the composite of intermediate goods purchased at time t from suppliers chosen $t-k$ periods ago satisfies $(P_{di,t}^{k})^{1-\sigma_j} = \int_{\omega\in\Omega_{i,t}^{k}} p(\omega)_{di,t}^{1-\sigma_j} \mathrm{d}\omega$. The sets $\left\{ \Omega_{i,t}^{k} \right\}_{k=0}^{\infty}$ form a partition of industry i's product space, so we can obtain country d's price index for industry i goods at time t by aggregating these partial price indices over all partitions and find $P_{di,t}^{1-\sigma_j} = \sum_{k=0}^{\infty} \left(P_{di,t}^{k} \right)^{1-\sigma_j}$.

We establish in Appendix B.2 that the partial price index for the set of intermediate goods whose suppliers are being chosen at time t takes the familiar form

$$P_{di,t}^{0} = \gamma_i \, \mu_{i,t}(0)^{1/(1-\sigma_j)} \left(\Phi_{di,t}^{0}\right)^{-\frac{1}{\theta_i}}, \tag{15}$$

where $\gamma_i \equiv \Gamma\left([\theta_i - \sigma_i + 1]/\theta_j\right)^{1-\sigma_j}$ is a constant, $\Phi_{di,t}^{0}$ is given by (10), and $\mu_{i,t}(0)$ denotes the measure of the set $\Omega_{i,t}^{0}$. Following the previous discussion, the endogenous market access term $\Phi_{di,t}^{0}$ represents the mean price of intermediate goods whose suppliers are chosen at time t. The measure $\mu_{i,t}(0)$ accounts for gains from variety. This measure recursively evolves over time according to the stochastic process that governs sourcing decisions, given by

$$\mu_{i,t}(k) = \begin{cases} \zeta_i, & k = 0 \\ (1-\zeta_i)\mu_{i,t-1}(k-1), & k > 0. \end{cases} \tag{16}$$

As we show in Appendix B.3, the partial price index across intermediate goods whose suppliers were last chosen at time $t-k$ is given by

$$P_{di,t}^{k} = P_{di,t-k}^{0} \left(\frac{\mu_{i,t}(k)}{\mu_{i,t-k}(0)} \Phi_{di,t}^{k}\right)^{1/(1-\sigma_i)}, k > 1 \tag{17}$$

which is the period $t-k$ price index of the basket of intermediate goods Ω_{t-k}^{0}, adjusted for the subsequent change in variety composition, captured by $\mu_{i,t}(k)/\mu_{i,t-k}(0)$, and prices, captured by $\Phi_{di,t}^{k}$.

Given Equations (15) and (17), we can solve for the composite price index of industry i goods in country d at time t:

$$P_{di,t} = \gamma_i \left(\Phi_{di,t}^{0}\right)^{-\frac{1}{\theta_i}} \left[\mu_{i,t}(0) + \sum_{k=1}^{\infty} \mu_{i,t}(k) \left(\frac{\Phi_{di,t}^{0}}{\Phi_{di,t-k}^{0}}\right)^{\frac{1-\sigma_i}{\theta_i}} \Phi_{di,t}^{k}\right]^{\frac{1}{1-\sigma_j}} \tag{18}$$

The term $\gamma_i \left(\Phi_{di,t}^{0}\right)^{-1/\theta_i}$ on the right-hand-side of Equation (18) captures the prices paid under flexible supplier choice. The term in brackets quantifies the extend to which current aggregate demand is affected by the stickiness of supply relationships. The terms $\Phi_{di,t}^{k}$ capture differences in demand across intermediate goods driven by differences in the age of their supply relationships and reflect their impact on aggregate demand at time t. The term $(\Phi_{di,t}^{0}/\Phi_{di,t-k}^{0})^{(1-\sigma_i)/\theta_i}$ measures the current demand of a buyer whose supplier relationship from k periods ago differs from that of a buyer who just updated its supplier.

Using the above price indices, we can readily derive country d's expenditure share on industry i goods sourced from country s

$$\lambda_{sdi,t} = \sum_{k=0}^{\infty} \lambda_{sdi,t}^{k} \left(\frac{P_{di,t}^{k}}{P_{di,t}} \right)^{1-\sigma_i}. \tag{19}$$

where $\lambda_{sdi,t}^{k}$ is given by Equation (11) if $k = 0$ and (13) if $k > 0$.

The set of trade shares $\{\lambda_{sdi,t}\}_{s,d\in\mathcal{N},i\in\mathcal{I}}$ fully characterize demand in the world economy at time t. To close the model, we now describe the conditions for market clearing and define a general equilibrium.

2.4 Equilibrium

Denote the total revenue of an industry i in a source country s at time t by $X_{si,t}$. To define equilibrium, we express each industry's revenue in terms of trade shares, given by Equation (19), and total expenditures on consumption, $E_{d,t}$, and intermediate inputs in the rest of the world:

$$X_{si,t} = \sum_{d\in\mathcal{N}} \lambda_{sdi,t} \left[\eta_{di} E_{d,t} + \sum_{j\in\mathcal{I}} \alpha_{sij} X_{dj,t} \right]. \tag{20}$$

A country's national consumption spending is the sum of its factor income and trade deficit, $E_{d,t} = w_{d,t} L_{d,t} + D_{d,t}$, with $\sum_{d\in\mathcal{N}} D_{d,t} = 0$. We follow the conventional approach in the international trade literature and treat aggregate trade deficits as exogenous. To clear the factor market, wages then adjust to ensure that expenditures equal disposable income,

$$w_{d,t} L_{d,t} = \sum_{i\in\mathcal{I}} (1 - \alpha_{di}) X_{di,t}, \tag{21}$$

and goods market clearing is guaranteed by Walras' law.

We are now ready to define a dynamic general equilibrium and a steady state.

Definition 1. *An economy is described by a set of time-invariant parameters summarizing technologies, preferences and factor endowments,* $\boldsymbol{\Theta} = \left\{ \theta_i, \sigma_i, \{\alpha_{dji}\}_{j\in\mathcal{I}}, \varphi_{di}, A_{di}, \eta_{di}, L_d \}_{d\in\mathcal{N}} \right\}_{i\in\mathcal{I}}$*, sourcing frictions* $\boldsymbol{\zeta} = \{\zeta_i\}_{i\in\mathcal{I}}$*, as well as a measure* $\boldsymbol{\mu}_{t_0} = \{\mu_{t_0}(k)\}_{k\in\{0,1,\cdots\}}$ *for some* t_0*. Given histories of trade costs* $\boldsymbol{\tau}_{t-1} \equiv \{\tau_t\}_{\varsigma<t} = \left\{ \tau_{sid,\varsigma} \right\}_{s,d\in\mathcal{N},i\in\mathcal{I},\varsigma<t}$ *and their changes* $\hat{\tau}_t \equiv \{\hat{\tau}_{sdi,t}\}_{s,d\in\mathcal{N},i\in\mathcal{I}}$ *as well as nominal wages* $\boldsymbol{w}_{t-1} = \{w_\varsigma\}_{\varsigma<t} = \{w_{d,\varsigma}\}_{d\in\mathcal{N},\varsigma<t}$*:*

1. *A static equilibrium at time* t *is a vector of wages* $w(\hat{\tau}_t \times \tau_{t-1} \cup \boldsymbol{\tau}_{t-1}, \boldsymbol{w}_{t-1}, \zeta, \Theta) = w_t$ *that jointly solves Equations (19) to (21) for all* $s, d \in \mathcal{N}$ *and* $i \in \mathcal{I}$*.*

2. *A dynamic equilibrium at time t is a history of wages $\boldsymbol{w}_t$ so that, for all $w_\varsigma \in \boldsymbol{w}_t$, $w_\varsigma = w(\hat{\tau}_{\varsigma-1} \times \tau_{\varsigma-1} \cup \boldsymbol{\tau}_{\varsigma-1}, w_{\varsigma-1} \cup \boldsymbol{w}_{\varsigma-2}, \zeta, \Theta)$.*

3. *A dynamic equilibrium at time t is a steady state if $w(\mathbf{1}_{N\times N\times I} \times \tau_t \cup \boldsymbol{\tau}_{t-1}, w_t \cup \boldsymbol{w}_{t-1}, \zeta, \Theta) = w_t$.*

2.5 Steady-State Properties

In the following, we show that our model preserves the class of quantitative trade models based on Eaton and Kortum (2002) in the limit when the economy is in steady state, irrespective of the magnitude of the frictions underlying imperfect supplier adjustment, $\zeta_i \in (0, 1)$. Intuitively, the transitory effects of trade disruptions that arise in our model reflect how opportunities for finding new suppliers are limited in the short-run but increasing over time. As assemblers get to adjust all supply relationships in the long-run, we then obtain the EK-model as the limit of the equilibria along the transition path.

More formally, let $w^{EK}(\hat{\tau}_t \times \tau_{t-1} \cup \boldsymbol{\tau}_{t-1}, \boldsymbol{w}_{t-1}, 1, \Theta)$ represent the equilibrium allocation in an economy in which suppliers can be flexibly adjusted for all goods, $\zeta_i = 1$ for all i. We can then establish

Proposition 1. *If w_{t^*} is a steady state equilibrium, then*

1. *For any ζ, $w_{t^*} = w(\mathbf{1}_{N\times N\times I} \times \tau_{t^*} \cup \boldsymbol{\tau}_{t^*-1}, w_{t^*} \cup \boldsymbol{w}_{t^*-1}, \zeta, \Theta) = w^{EK}(\mathbf{1}_{N\times N\times I} \times \tau_{t^*} \cup \boldsymbol{\tau}_{t^*-1}, w_{t^*} \cup \boldsymbol{w}_{t^*-1}, 1, \Theta)$.*

2. *For all $k \in \{0, 1, ...\}$, the measure of goods $\omega \in \Omega^k_{i,t}$ equals $\mu_{i,t^*}(k) = (1 - \zeta_i)^k \zeta_i$, and trade flows are given by $\lambda^k_{sdi,t^*} = \lambda_{sdi,t} = \lambda^{EK}_{sid}$ where λ^{EK}_{sid} denotes the trade shares in the frictionless economy.*

Proposition 1 provides numerous useful insights. The first part makes clear that the tools developed by the literature studying the equilibrium properties of static quantitative trade models can be deployed to establish the existence and uniqueness of steady states in our model.

The second part of Proposition 1 highlights properties of the steady states that we later leverage to quantify the model. In particular, it shows that the process governing the evolution of the age distribution of supply relationships over time has a simple geometric stationary distribution. Further, it shows that steady state expenditure allocations are equalized across goods within an industry, irrespective of when their supplier was chosen.

3 Dynamic Adjustment to Trade Shocks

In this section, we theoretically characterize the economy's dynamic response to trade disruptions. In particular, we derive a new structural estimating equation for the trade elasticity at different time horizons, and show that transitional dynamics can be characterized using the dynamic hat-algebra. Finally, we provide a new formula for characterizing the horizon-specific gains from trade.

3.1 Trade Elasticity by Time Horizon

We begin by showing how the trade elasticity, that is the elasticity of trade flows with respect to transport cost, varies over time. To do so, we let $\varepsilon^h_{sdi,t}$ denote the trade elasticity at horizon h, which we define by:

$$\varepsilon^h_{sdi,t-1} \equiv \left.\frac{\partial \log X_{sdi,t+h}}{\partial \log \tau_{sdi,t}}\right|_{\{\Phi^k_{di,t+\varsigma}\}_{t\leq\varsigma\leq h,k}}, \tag{22}$$

which is the elasticity of trade flows in industry i from country s to d at time $t+h$, $X_{sdi,t+h}/X_{sdi,t-1}$ with respect to change in trade costs at t, $\mathrm{d}\log\tau_{sdi,t} = \log\hat{\tau}_{sdi,t}$, holding fixed the general equilibrium terms that summarize changes in market access for industry i goods in destination d. The following derives a closed-form expression for this elasticity.

Proposition 2. *Suppose that the economy is in steady state at $t=-1$. Then, up to a first order, the horizon-h response of trade flows to a shock to trade cost at time $t=0$ is given by:*

$$\varepsilon^h_i = -\theta_i\left[1-(1-\zeta_i)^{h+1}\right] - (\sigma_i - 1)(1-\zeta_i)^{h+1}. \tag{23}$$

If $\zeta_i \in (0,1)$, $\lim_{h\to\infty}\varepsilon^h_i = -\theta_i$, where the rate of convergence equals

$$\lim_{h\to\infty}\frac{\varepsilon^{h+1}_j + \theta_j}{\varepsilon^h_i + \theta_i} = \log(1-\zeta_i).$$

Following Proposition 2, the trade elasticity increases over time if $\theta_i > \sigma_i - 1$. In the long-run, it is equal to the Fréchet parameter θ_i, where the rate of convergence, intuitively, depends on the frequency at which buyers can establish a new sourcing relationship ζ_i.

It is worth noting that Equation (22) is consistent with reduced-form estimates of the trade elasticity at varying time horizons as in Boehm, Levchenko and Pandalai-Nayar (2023). Later, we leverage this equivalence to identify the key structural parameters in our model. The horizon-specific formulation of the trade elasticity implied by our model also induces a horizon-specific welfare formula, which we provide next.

3.2 The Horizon-Specific Welfare Gains from Trade

When supply relationships are slow to adjust to shocks, trade disruptions can put the economy through a sustained period of readjustment. The following proposition shows that our framework yields a simple formula for welfare analysis, giving changes in real wages associated with an initial set of foreign shocks over varying time horizons.

Proposition 3. *Suppose the economy is in steady state at $t = -1$. Then, the change in real wages in country d at time $h = \{0, 1, ...\}$, $\hat{W}_d^h = C_{d,h}/C_{d,-1}$, that follows a set of arbitrary shocks to trade cost at time at $t = 0$, is given by*

$$\hat{W}_d^h = \prod_{i,j\in\mathcal{I}} \left[\left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}} \right)^{-\frac{1}{\theta_j}} \left(\Xi_{dj,h} \right)^{\frac{1}{\sigma_j - 1}} \right]^{\eta_i \bar{a}_{dji}}, \tag{24}$$

where

$$\Xi_{dj,h} \equiv \zeta_j \left(\frac{\lambda_{ddj,h}}{\lambda^{k=0}_{ddj,h}} \right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}} + (1-\zeta_j)^{h+1} \left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}} \right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}} + \sum_{\varsigma=1}^{h} \zeta_j (1-\zeta_j)^k \left(\frac{\lambda_{ddj,h}}{\lambda^{k=0}_{ddj,h-\varsigma}} \right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}}, \tag{25}$$

and $\bar{a}_{dji}$ is the (j, i)-th element of the Leontief inverse $(\boldsymbol{I}d - \boldsymbol{A}_d)^{-1}$, with the elements of $\boldsymbol{A}_d$ given by α_{dji}. If $\zeta_i \in (0, 1)$, then $\lim_{h\to\infty} \hat{W}_d^h = \lim_{h\to\infty} \prod_{i,j\in\mathcal{I}} \left(\lambda_{ddj,t+h}/\lambda_{ddj,-1}\right)^{-\eta_i \bar{a}_{dji}/\theta_j}$.

Although our model features transition dynamics on the supply side, Equation (24) shows that welfare analysis can still be conducted using only a few sufficient statistics. These statistics delineate how the impact of trade shocks on real wages varies over time due to staggered sourcing decisions, decomposing the change in real wages associated with foreign shocks into two effects.

The first effect is captured by the terms $(\lambda_{ddj,t+h}/\lambda_{ddj,-1})^{-1/\theta_j}$ on the right-hand-side of Equation (24). Because the Fréchet parameter θ_j gives the price elasticity of trade flows sourced from the currently cheapest global supplier and the share of domestic expenditures the response of trade to prices, each of these terms would give the change in a particular industry j's domestic price index if all goods were optimally sourced. Because all supply relationships are flexible in the long-run, i.e., when $h \to \infty$, changes in aggregate home expenditure shares and the long-run trade elasticity, thus, remain sufficient for long-run welfare analysis in our model, as in Eaton and Kortum (2002). However, staggered sourcing decisions spell additional welfare effects in the short-run, i.e., when not all goods can be sourced optimally.

Staggered adjustment of suppliers spells time-varying distortions in prices and terms-of-trade, captured by the terms $(\Xi_{dj,h})^{1/(\sigma_j-1)}$ in Equation (24). Intuitively, these distortions manifest via expenditure allocations, and will vary across goods depending on when their current supplier was chosen. If a good was last optimally sourced k periods ago, the resulting distortion in its price at horizon h can be informed by the difference between the share of domestic expenditures on all goods time h and on optimally sourced goods at time $h-k$, $(\lambda_{ddj,h}/\lambda^{k=0}_{ddj,h-k})^{(\sigma_j-1-\theta_j)/\theta_j}$. Intuitively, a decrease in $\lambda_{ddj,h}/\lambda^{k=0}_{ddj,h-k}$ indicates that suppliers that were chosen k periods ago are now, at horizon h, less competitive; the implied deterioration in a country's aggregate terms-of-trade is decreasing in the elasticity of substitution, σ_j, and increasing in the share of goods sourced from these suppliers, $\zeta_j \cdot (1-\zeta_j)^k$, is higher.

As an implication of Proposition 3, the trade elasticity relevant for welfare analysis varies over time. To further illustrate this point, it is useful to approximate changes in industry-level prices up to a first-order, which yields

$$\log(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}})^{-\frac{1}{\theta_j}}(\Xi_{dj,h})^{\frac{1}{\sigma_j-1}} \approx -\frac{1}{\theta_j}[1-(1-\zeta_j)^{h+1}]\log\frac{\lambda^{k=0}_{ddj,h}}{\lambda_{ddj,-1}} - \frac{1}{\sigma_j-1}(1-\zeta_j)^{h+1}\log\frac{\lambda^{k=h+1}_{ddj,h}}{\lambda_{ddj,-1}} - \mathcal{E}^h_{dj},$$

where $\mathcal{E}^h_{dj} = \sum_{\varsigma=1}^{h+1}(1-\zeta)^{\varsigma}\zeta\left[\frac{1}{\sigma_j-1}\log\frac{\lambda^{k=\varsigma}_{ddj,h}}{\lambda^{k=0}_{ddj,h-\varsigma_j+1}} - \frac{1}{\theta_j}\log\frac{\lambda^{k=0}_{ddj,h+1-\varsigma}}{\lambda^{0}_{ddj,-1}}\right]$.

The first term on the right captures how changes in the prices of goods that were procured optimally at least once contribute to the overall change in prices at horizon h, assuming that past changes in factor prices were equal to those observed h periods after the shock. The second term, in contrast, captures changes in aggregate prices due to changes in the prices of goods whose suppliers have never been adjusted. The relative importance of these two effects varies over time, in tandem with the structural trade elasticity.

The last term, $\mathcal{E}^h_{dj}$, captures how suboptimal sourcing decisions from the past continue to distort prices at horizon h by distorting the equilibrium adjustment of factor prices relative to the long-run. Such distortions are reflected in price differences between goods whose suppliers were adjusted before and those that are procured optimally at horizon h.

Staggered sourcing decisions, hence, imply that the trade elasticity relevant for welfare analysis differs from the structural elasticity in Equation (23) due the dynamic interaction of sourcing decisions and factor prices. Due to these interactions, the welfare effects of trade shocks may, then, vary both quantitatively and qualitatively over time, even conditional on the structural parameters underlying the time variation in the trade elasticity. Viewed through this lens, Proposition 3 is fortunate in that it allows us to summarize these dynamic effects in terms of a few statistics, which, as we will now describe, also enables us deploy familiar tools from the international trade literature to solve exactly for the equilibrium response of prices and wages to trade shocks implied by the model.

3.3 Characterization of Impulse Responses

We now show that solving for the responses of trade and production to shocks does not require knowledge of the economy's structural fundamentals (productivities, and trade costs). As an implication, the so-called "hat algebra" of Dekle, Eaton and Kortum (2007) can be deployed to characterize impulse responses in our model.

Trade flows at time t can be expressed in terms of succinct changes in trade costs and wages, as well as past changes in trade flows for optimally sourced goods, trade costs and wages:

$$\lambda_{sdi,t} = \frac{\left[1 + \left(\hat{\tau}_{sdi,t}\hat{w}_{s,t}/\hat{w}_{d,t}\right)^{1-\sigma_i+\theta_i} \omega_{sdi,t-1}\right] \lambda^{k=0}_{sdi,t-1} \left(\hat{\tau}_{sdi,t}\hat{w}_{s,t}\right)^{-\theta_i}}{\sum_{s'\in\mathcal{N}} \left[1 + \left(\hat{\tau}_{s'id,t}\hat{w}_{s',t}/\hat{w}_{d,t}\right)^{1-\sigma_i+\theta_i} \omega_{s'id,t-1}\right] \lambda^{k=0}_{s'id,t-1} \left(\hat{\tau}_{s'id,t}\hat{w}_{s',t}\right)^{-\theta_i}}, \tag{26}$$

where the wedges

$$\omega_{sdi,t-1} \equiv \frac{\mu_{i,t}(1)}{\mu_{i,t}(0)} + \sum_{k'=2}^{\infty} \frac{\mu_{i,t}(k')}{\mu_{i,t}(0)} \left(\frac{\lambda^{k=0}_{ddi,t-1}}{\lambda^{k=0}_{ddi,t-k'}}\right)^{\frac{\sigma_i-1}{\theta_i}} \frac{\lambda^{k=k'}_{sdi,t-1}}{\lambda^{k=0}_{sdi,t-1}} \prod_{\varsigma=t-k''+1}^{t-1} \left(\hat{\tau}_{sid,\varsigma}\frac{\hat{w}_{s,t}}{\hat{w}_{d,\varsigma}}\right)^{1-\sigma_i}, \tag{27}$$

summarize how prior distortions in factor prices continue to impact trade flows at time t by distorting the terms of trade.

Now suppose that the economy was in steady state at some time prior to t. Then, given bilateral country-sector trade flows, industry-level consumption and intermediate good expenditure shares as well as per-capita GDP, the only additional industry-level parameters that are required to recursively compute changes in trade flows at increasing time horizons are given by $\{\zeta_i, \theta_i, \sigma_i\}$. Given this recursive formulation for trade flows, we can express the market clearing conditions (21) in terms of changes in trade costs and factor prices, as in Dekle, Eaton and Kortum (2007), and, hence, solve for the period-by-period change in wages associated with (a sequence of) trade shocks.

4 Estimation

We now turn to exploring the quantitative implications of our theory for the response of production and welfare to trade shocks. In this section, we outline and implement our approach to estimating the structural parameters that govern the time variation of the trade elasticity. In the next section, we will use these estimates to provide a quantitative assessment ramifications of the 2018 US-China trade war for trade, production and welfare.

4.1 Approach

Proposition 2 implies that we can express the trade elasticity at varying time horizons h as a function of the set of structural parameters $\Theta_i \equiv \{\theta_i, \sigma_i, \zeta_i\}$:

$$f_i^h(\Theta_i) \equiv \varepsilon_i^h = \frac{\partial \log \lambda_{sdi,t}}{\partial \log \tau_{sid,0}} = -\theta_i \left[1 - (1-\zeta_i)^{h+1}\right] + (1-\sigma_i)(1-\zeta_i)^{h+1}.$$

Our approach to recovering these structural involves, as a first step, obtaining reduced-form estimates of the trade elasticity over varying horizons. Such estimates can be obtained from the following specification using local projection methods:

$$\log\left(\frac{X_{sdi,t+h}}{X_{sdi,t-1}}\right) = \beta_i^h \log\left(\frac{\bar{\tau}_{sdi,t}}{\bar{\tau}_{sdi,t-1}}\right) + \delta_{si,t+h} + \delta_{di,t+h} + u_{sdi,t+h},$$

where $X_{sdi,t}$ denotes the exports of industry i goods from s to d at time t,and $t_{sdi,t}$ is the associated gross ad valorem tariff. The remaining terms denote source- or destination-industry-year-specific country fixed effects, and $u_{sdi,t}$ is an idiosyncratic error term. The coefficient β_X^h captures the change in trade flows h periods ahead that follows an initial one-period change in tariffs. Suppose that tariff changes were always one-time permanent shocks. Then a consistent estimate of β_i^h would yield an estimate of the structural trade elasticity at horizon h, ε_i^h. We now show how to recover the structural parameters governing the trade elasticity in our model, given a set of reduced-form estimates its behavior at varying time horizons h. With a slight abuse of notation, let $\{\hat{\beta}_i^h\}_{h=0}^H$ denote a set of such estimates ranging up to horizon $H > 0$.

Intuitively, the parameter σ_i governs the behavior of the trade elasticity in the short-run, while θ_i pins down its long-run value. The rate at which the trade elasticity converges to its long-run value, in turn, depends on how fast buyers form new supply relationships, ζ_i. More formally, we can use the structural

Table 1: Trade Elasticity Parameter Estimates for the Manufacturing Industry

Parameter		Estimate
Supplier adjustment probability	ζ	0.09
Long-run Trade Elasticity	θ	3.16
Short-run Trade Elasticity	$\sigma - 1$	0.11

expression for the trade elasticity to show that ζ_i, at any time $h > 0$, satisfies

$$\log(1 - \zeta_i) = \frac{1}{h} \log \left(\frac{f_i^H(\Theta) - \theta_i}{f_i^0(\Theta) - \theta_i} \right), \tag{28}$$

which captures the rate at which the process governing the trade elasticity converges to its long-run limit. Given a set of reduced-form estimates $\hat{\boldsymbol{\beta}}_i \equiv \{\hat{\beta}_i^h\}_{h=0}^H$, we recover our structural parameters by minimum distance:

$$\hat{\Theta}_i(\hat{\boldsymbol{\beta}}_i) = \arg\min_{\Theta} (f_i^h(\Theta) - \hat{\beta}_i^h)_{i \in \mathcal{I}})^T W \left(f_i^h(\Theta) - \hat{\beta}_i^h \right)_{i \in \mathcal{I}}, \tag{29}$$

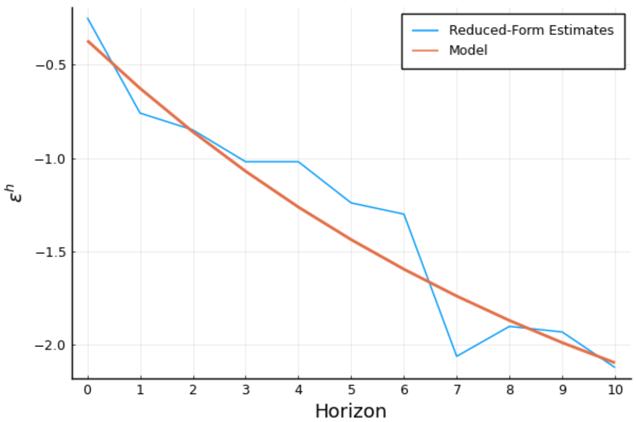
where W is a H-dimensional weighting matrix. Provided that the estimates of the trade elasticity are consistent, the continuous mapping theorem implies that $\hat{\Theta}_i(\hat{\boldsymbol{\beta}}_i)$ will provide a consistent estimate of Θ.

4.2 Implementation and Results

To implement our estimation approach, we leverage a set of comprehensive reduced-form estimates of the trade elasticity at different time horizons by Boehm, Levchenko and Pandalai-Nayar (2023). Following the reduced-form empirical approach outlined above, they find that arguably exogenous tariff changes in third countries predict a short-run trade elasticity that is substantially lower over shorter compared to longer horizons h, where $h = 0, 1, ..., 10$. To recover our set of structural parameters, we focus on matching the implied empirical behavior of the trade elasticity within the first two years, as well as at horizons $h = \{8, 9, 10\}$. Specifically, we set the weighting matrix W so that our estimator targets the response of trade flows to an initial change in tariffs Table 1 presents the results.

We find that supply relationships reset at an annual rate of about 9 percent, indicating substantial stickiness in supply relationships. The long-run trade elasticity across manufacturing industries, on average, equals 3.2, consistent with estimates in the literature on gravity. Our estimate of the elasticity of substitution equals 1.145, suggesting that trade elasticity, in the short-run, will be substantially lower, given the stickiness of supply relationships.

Figure 1: The trade elasticity at varying time horizons



Notes: The red line depicts the structural trade elasticity in Equation (23) under the parameter estimates displayed in Table 1. The blue line plots the corresponding reduced-form estimates by Boehm, Flaaen and Pandalai-Nayar (2023).

Figure 1 graphs the structural trade elasticity implied by these parameter estimates, along with the reduced-form elasticity estimates by Boehm, Levchenko and Pandalai-Nayar (2023). On impact ($h = 0$), the structural trade elasticity is close to zero. Over time, it smoothly increases in absolute value, reflecting the gradual resetting of supply relationship and reaching a level of -2.2 after 10 years. Reassuringly, the structural trade elasticity matches the behavior of its empirical counterpart also at horizons that were not explicitly targeted by our estimator.

To explore heterogeneity of trade elasticities across different industries, we implement our approach to separately estimate parameters for 10 distinct H.S. product categories, utilizing the corresponding reduced-form estimates found in Boehm, Flaaen and Pandalai-Nayar (2023). Our results, as shown in Table 2, reveal significant variation in these parameters across industries. Across industries, the parameter estimates for the rate of supplier adjustment ζ_i range from as low as 2 percent to 36 percent, while those for the Fréchet parameter θ_i indicate values of long-run trade elasticities range from 1.8 and 7.2. The parameter estimates for σ_i, in comparison, vary little across sectors. This variation in parameter estimates has implications for the magnitudes of short- and long-run trade elasticities, governed by σ_i and θ_i, and the rate at which trade elasticities converge to their long-run value, governed by ζ_i, as well as for welfare analysis, following Proposition 3.

5 Quantitative Application: The US-China Trade War

Armed with our structural estimates, we now apply our model to study the general equilibrium response of trade and production to the US-China trade war.

Table 2: Trade Elasticity Parameter Estimates across H.S. product categories

Industry	Adjustment rate ζ_i	LR trade elasticity θ_i	SR trade elasticity σ_i
Plastics	0.08	1.8	1.44
Leather	0.09	7.2	0.9
Wood	0.15	3.9	1.5
Paper	0.18	3.3	0.4
Textile	0.29	3.5	0.5
Stone	0.12	5.9	0.4
Base Metals	0.08	3.6	1.2
Machinery	0.09	1.8	1.3
Optical Instruments	0.02	3.4	1.3
Others	0.36	3.8	1.6

5.1 Steady-State Calibration

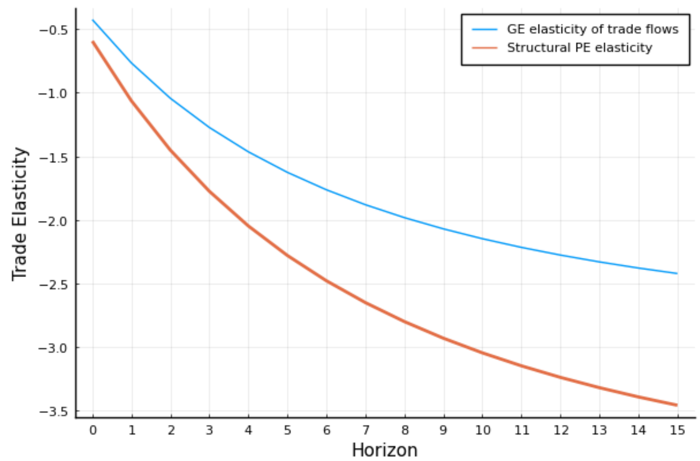
We compare the country-level aggregate outcomes after the rise in tariffs between the United States and China in 2018 with the outcomes in the absence of the U.S.-China trade war. To do so, we calibrate the initial steady state to the observed trade flows in 2017.[4] With the structural estimates from the previous section at hand, we, additionally, require information on aggregate trade flows, domestic production and expenditures, by country and industry, to calibrate our model to the initial steady state.

Trade and Production Data Our calibration for the initial steady state is based on information on trade flows that involve 44 countries (including a location for the "rest of the world") and 170 industries based on the 2016 release of the World Input-Output Database (Timmer et al., 2015) and the International Trade and Production Database for Estimation 2020 (Borchert et al., 2020). For consistency, we aggregate industries to match the eleven HS categories for which we obtained estimates of the structural trade parameters. We further collapse industries whose goods are not subject to tariffs into an aggregate services sector. In our baseline calibration, we assume that the services sector is not subject to adjustment frictions, rendering the elasticity of substitution in this industry irrelevant for counterfactual analysis. In keeping with the best practice in the literature, we assign its long-run trade elasticity a value equal to 4.

Tariffs We measure the tariff implications of the trade war by constructing import-weighted averages of the tariff changes documented by Fajgelbaum et al. (2020). The resulting set of one-period shocks raises trade costs between the US and China between 2 and 13 percent across industries.

[4] We could, alternatively, choose another year as our initial steady state, and then solve for the exact changes in trade costs and technologies that rationalize the trade flows in the year before the tariff escalations took place.

Figure 2: US-China Trade War Counterfactual: Equilibrium trade elasticity of bilateral US-CHN trade flows



Notes: This figure displays the counterfactual elasticity of bilateral trade flows between the US and China at horizon h that follows tariff increases in 2018.

5.2 Quantitative Results

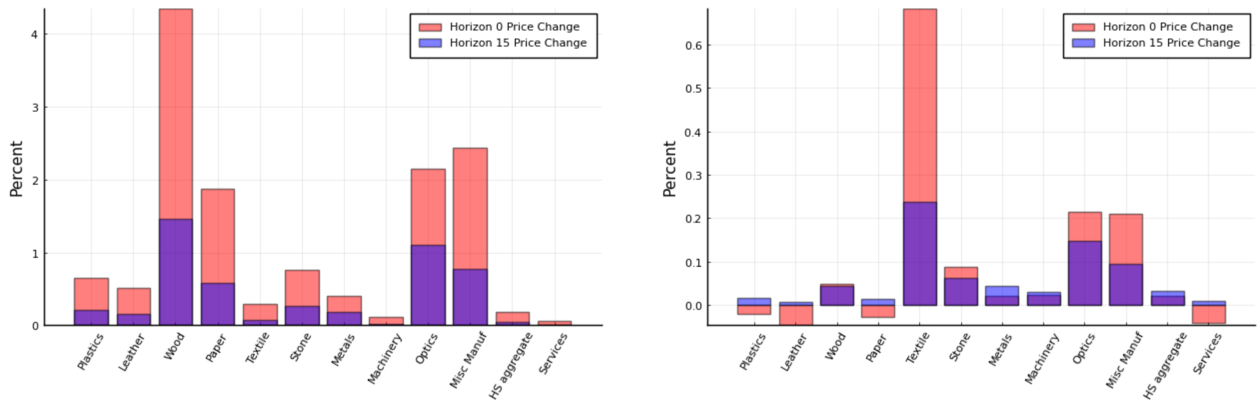
We simulate the dynamic general equilibrium responses of trade, production and welfare in all 44 countries to the initial changes in trade costs between the US and China in 2018. Before describing the normative implications of the trade war for US welfare, we first show how trade flows and prices adjust in response to the shock.

Trade flows As expected, bilateral trade between China and the US falls in response to the rise in tariffs. As shown in Figure 2, the implied total trade elasticity, averaged across non-services industries, increases in absolute value over time until it settles at a long-run value of about 2. In the short-run, the responsiveness of trade flows to price shocks is well approximated by the partial trade elasticity in (23), highlighting how factor prices, initially, respond too little to trade disruptions due to the sluggishness of short-run demand. As this demand gradually adjusts over time, bilateral exports continue to fall; however, trade flows fall by less than predicted by the structural trade elasticity due to simultaneous adjustments in world factor prices.

Prices. The sluggish short-run response of US demand to the rise in trade costs induces a substantial rise in its domestic price level. As shown in Figure 3, aggregate output prices rise across all industries in the U.S, where some industries see prices rise by over 4% upon impact. As sourcing decisions gradually adjust to the initial rise in trade cost, over half of this initial hike in prices will be undone 15 periods after the shock.

Figure 3: Trade War Counterfactual: Changes in domestic output prices by industry

(a) US (b) China



Notes: This figure displays the counterfactual changes in aggregate price indices for each industry in the US and China at horizons $h = 0$ and $h = 15$.

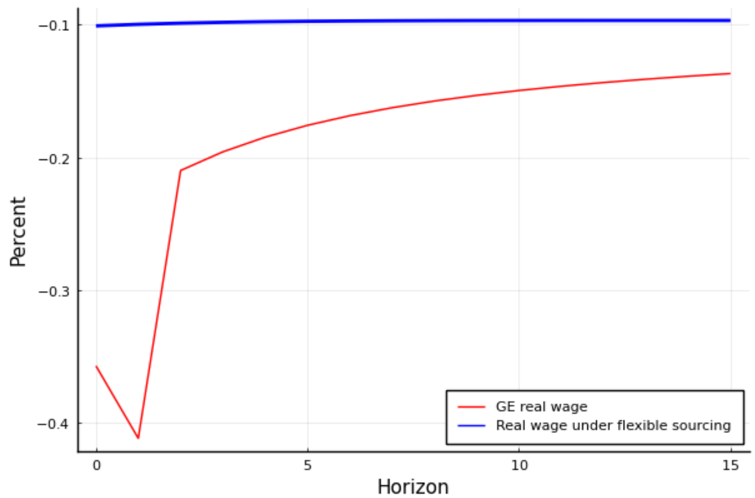
In contrast to domestic prices in the US, the responses of domestic prices in China vary by industry and differ over time both quantitatively and qualitatively. As in the US, the increase in tariffs results in higher output prices for Chinese consumers in the long-run. In the short-run, however, some industries in China see a decline in domestic output prices, for example, plastics, leather, and services. Intuitively, a rise in trade barriers can temporarily improve a country's terms-of-trade when trade is not primarily driven by comparative advantage.

Real Wages and Welfare. Figure 5 traces the horizon-specific responses of the real wage in the US to the trade war. The red line displays the responses of the US's real wage over the $h = 0, 1, ...$ periods that follow the initial onset of the trade war in 2018, $h = 0$, relative to steady state. To account for the importance of sourcing frictions for these welfare changes, we also graph, in blue, the hypothetical change in welfare if new suppliers could be found instantly, given realized changes in factor prices (following Proposition 2).

In the US, the onset of the trade war leads to a decline of its real wage equal to -0.35%, reflecting the previously discussed spike in domestic output prices. Frictions in sourcing decisions account for more than 75% of this initial decline in real wages. In particular, our decomposition shows that real wages would have only decreased by about 0.1% if all supply relationships could have flexibly adjusted, holding fixed the changes in factor prices.

The transitory dynamics of real wages in the periods following the arrival of the shock reflect the interaction of two opposing forces. On the one hand, as a growing number of supply relationships get to be reset, the magnitude of price distortions decreases over time; on the other hand, they compound for the subset of goods that continue to be sourced from previous suppliers. Figure 2 shows that the

Figure 4: US-China Trade War Counterfactual: Response of Real Wages in the US



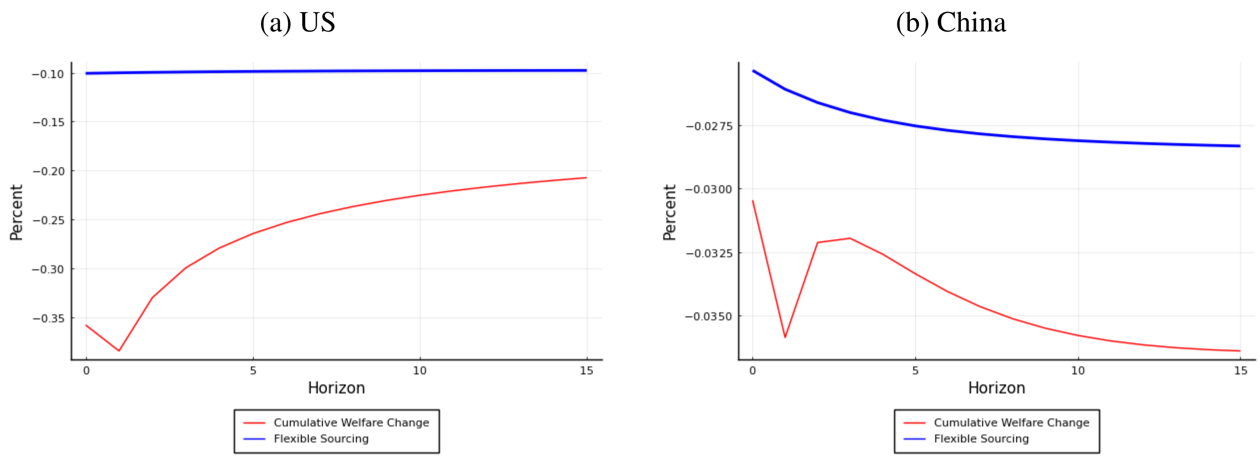
latter initially dominates the former effect, resulting the real wage in the US to decline further in 2019 (horizon $h = 1$), falling -0.4 percent below its initial value. Price distortions only start to decline from horizon $h = 2$ onward, resulting in a gradual rise of the real wage. The real wage will then continue to increase up until it converges with that implied by the long-run model.

To quantify the welfare loss associated with these transitional dynamics in real wages, we adopt consumption equivalence as a welfare measure, assuming that consumers have logarithmic inter-temporal preferences, $u(C) = \log C$ and a discount future utility flows at a rate $\beta = 0.95$. Given these assumptions, we then separately calculate the consumption equivalent welfare change corresponding to a given counterfactual path of real wages at each time horizon $h = 0, 1, 2, ...$

Figure 6 displays the results, showing the cumulative welfare effects of the trade war on the US and China. The response of US welfare shows that the dynamic costs of trade disruptions are about 70% higher than those implied by steady-state comparisons: While the long-run effects of the trade war induce a consumption-equivalent welfare loss of 0.1%, the transitional supply disruptions that play out over the short-run amount to a welfare loss of 0.17%.

The right panel in figure 2 shows that the welfare costs of the trade war born by China are an order of magnitude smaller than those in the US. The overall consumption-equivalent welfare losses in China is equal to -0.033%, compared to a steady state welfare loss of -0.028%. As a key point of departure, we find that price distortions primarily affect welfare through current supplier choices, rather than through the prices of old contracts. That is, transitory welfare losses, in China, primarily result from the interaction of short-run distortions in world factor prices and current sourcing decisions.

Figure 5: Trade War Counterfactual: Consumption Equivalent Welfare Changes



Notes: Cumulative response of consumption equivalent welfare to an initial rise in tariffs in period $h = 0$, assuming logarithmic intertemporal utility and a discount factor of $\beta = 0.95$. Total welfare effect is displayed in red. The blue line shows the change in welfare due to adjustments domestic trade shares for optimally sourced goods, following Proposition 2.

We conclude by briefly discussing the welfare implications for countries not directly impacted by the trade war, which we summarize in table 4. We find that the short-run effects of trade disruptions negatively impact welfare across all countries, where losses are intuitively concentrated in major trading partners of either the US or China, such as Mexico, Canada, or Japan. However, while short-run welfare losses among third-party countries are largest in Mexico in the initial periods following the trade disruption, Mexico also stands out as one of the few countries that gains from the trade war in the long-run. Intuitively, in the short-run, price disruptions propagate through the international trade network and negatively impact all countries. In the long-run, however, the readjustment of supply relationships that follows a trade disruption can be beneficial for some countries. Mexico, in particular, benefits by increasing its exports to the US, while becoming a major export destination for Chinese goods.

6 Concluding Remarks

To account for imperfect adjustment to global supply-chain shocks, we develop a Ricardian trade framework with frictions that result in infrequent decisions of producers to change global suppliers. We obtain novel formulas for welfare changes to trade openness and trade shocks, derive novel estimation equations for trade elasticity estimation at varying time horizons, and quantify the model. Simulations of the so-called China-US trade war episode suggest that rich sectoral dynamics ensue, resulting in considerable short-term reallocations and substantive welfare fluctuations.

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Appendix

A Additional Tables

Table 3: Quantification of the model

Industry	Parameters				
	ζ_i	θ_i	σ_i	$\hat{\tau}_{US,CHN}$	$\hat{\tau}_{CHN,US}$
Plastics	0.08	1.8	1.44	1.11	1.11
Leather	0.09	7.2	0.9	1.10	1.10
Wood	0.15	3.9	1.5	1.14	1.14
Paper	0.18	3.3	0.4	1.10	1.10
Textile	0.29	3.5	0.5	1.10	1.10
Stone	0.115	5.9	0.4	1.18	1.18
Base Metals	0.08	3.6	1.2	1.17	1.10
Machinery	0.09	1.8	1.3	1.10	1.09
Optical Instruments	0.02	3.4	1.3	1.09	1.18
Misc Manufacturing	0.36	3.8	1.6	1.06	1.05
HS Aggregate	0.09	3.16	1.14		1.06
Services	1	4	-	1.03	1.02

Table 4: The U.S.-China Trade War: Counterfactual Welfare Changes in Selected Countries

Country	Cumulative Welfare Change (%)		
	$h=0$	$h=10$	$h\to\infty$
United States	-0.35	-0.23	-0.18
China	-0.03	-0.03	-0.04
Canada	-0.03	-0.02	-0.003
Mexico	-0.07	-0.02	0.01
Japan	-0.02	-0.004	0.004
Korea	-0.01	-0.002	0.000
Taiwan	-0.006	-0.0015	0.000
India	-0.009	-0.005	-0.000
UK	-0.014	-0.0065	-0.01
Germany	-0.012	-0.005	-0.001
France	-0.012	-0.006	0.001

Notes: Cumulative welfare changes of country i at horizon h: present discounted change in real wages over the h periods that follow upon an initial rise in trade costs between the US and China

B Equilibrium

B.1 Ideal Price Indexes and Generic Trade Shares

The composite good in industry j is

$$Y_{dj,t} \equiv \left(\int_{[0,1]} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}} .$$

Product space $\Omega_j = [0,1]$ can be partitioned into disjoint sets with $\Omega_j = \bigcup_{k=0}^{\infty} \Omega_{j,t}^k$, so we can rewrite the composite good as

$$Y_{dj,t} \equiv \left(\sum_{k=0}^{\infty} \int_{\Omega_{j,t}^k} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}} . \tag{B.1}$$

The assembler's associated cost minimization problem is

$$\begin{aligned}
\min_{\{y_{dj,t}(\bar{\omega})\}_{\bar{\omega}\in\Omega_{j,t}},\{Y^k_{dj,t}\}} P_{dj,t}Y_{dj,t} &= \sum_{k=0}^{\infty} P^k_{dj,t}Y^k_{dj,t} \\
s.t. \qquad Y_{dj,t} &= \left(\sum_{k=0}^{\infty}\left(Y^k_{dj,t}\right)^{\frac{\sigma_j-1}{\sigma_j}}\right)^{\frac{\sigma_j}{\sigma_j-1}}, Y^k_{dj,t} \equiv \left(\int_{\Omega^k_{j,t}} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j-1}{\sigma_j}}\mathrm{d}\bar{\omega}\right)^{\frac{\sigma_j}{\sigma_j-1}}, \\
P^k_{dj,t}Y^k_{dj,t} &= \int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})y_{dj,t}(\bar{\omega})\mathrm{d}\bar{\omega},
\end{aligned}$$

where we define the partial composite good $Y^k_{dj,t} \equiv \left(\int_{\Omega^k_{j,t}} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j-1}{\sigma_j}}\mathrm{d}\bar{\omega}\right)^{\frac{\sigma_j}{\sigma_j-1}}$ for each partition k as a helpful construct for derivations and implicity define the associated partial ideal price index $P^k_{dj,t}$ that satisfies $P^k_{dj,t}Y^k_{dj,t} = \int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})y_{dj,t}(\bar{\omega})\mathrm{d}\bar{\omega}$.

Under homotheticity of the assembler's production, this problem can be solved in two steps. First, the assembler decides which share of cost it allocates to each partial composite good $Y^k_{dj,t}$. Given those choices, the assembler then decides the optimal cost for each intermediate good $y_{dj,t}(\bar{\omega})$. Optimal demand satisfies

$$Y^k_{dj,t} = \left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{-\sigma_j} Y_{dj,t} \quad \text{and} \tag{B.2}$$

$$y^k_{dj,t}(\bar{\omega}) = \left(\frac{p_{dj,t}(\bar{\omega})}{P^k_{dj,t}}\right)^{-\sigma_j} Y^k_{dj,t} = \left(\frac{p_{dj,t}(\bar{\omega})}{P_{dj,t}}\right)^{-\sigma_j} Y_{dj,t} \quad \text{for each } \bar{\omega} \in \Omega^k_{j,t}, \tag{B.3}$$

where the last equality also shows that the partitioned solution equals the standard solution under a constant elasticity of substitution. Replacing the demand functions above in the definition of the budget constraint results in the expressions for the ideal price indices:

$$P_{dj,t} = \left(\int_{[0,1]} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega}\right)^{\frac{1}{1-\sigma_j}}, \qquad P^k_{dj,t} = \left(\int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega}\right)^{\frac{1}{1-\sigma_j}}. \tag{B.4}$$

We have now established that partitioning the product space into disjoint sets results in well-behaved demand functions such that, given optimal choices within each set, we can analyze demand for each intermediate good independently and then aggregate. In subsequent derivations, expenditure shares within each partition k will play a crucial role, so we state a general definition here:

$$\lambda_{sdj,t}^{k} \equiv \frac{X_{sdj,t}^{k}}{X_{dj,t}^{k}} \equiv \frac{\int_{\Omega_{j,t}^{k}} \mathbf{1}\left\{s \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}{\int_{\Omega_{j,t}^{k}} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega} = \frac{\int_{\Omega_{j,t}^{k}} \mathbf{1}\left\{s \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}{\sum_n \int_{\Omega_{j,t}^{k}} \mathbf{1}\left\{n \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}. \tag{B.5}$$

B.2 Trade Shares When Firms Are Sourcing Optimally ($k = 0$)

Under perfect competition, the destination price for intermediate good $\omega \in \Omega_{j,t}^{0}$ offered by country s to country d is $p_{sdj,t}(\omega) = c_{sdj,t}/z_{sj}(\omega)$ for the common unit cost component $c_{sdj,t}$ by (3) and supplier ω's productivity $z_{si}(\omega)$. Under the EK assumptions, the cumulative distribution function of prices is therefore

$$\tilde{F}_{sdj,t}(p) = \mathbb{P}\left[p_{sdj,t}(\omega) < p\right] = 1 - F_{sj}\left(\frac{c_{sdj,t}}{p}\right) = 1 - \exp\left\{-A_{sj}(c_{sdj,t})^{-\theta_j} p^{\theta_j}\right\}. \tag{B.6}$$

The resulting probability that country d sources an intermediate good $\omega \in \Omega_{j,t}^{0}$ from country s is

$$\mathbb{P}\left[s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] = \int_0^\infty \prod_{n \neq s} \left[1 - \tilde{F}_{ndj,t}(p)\right] \mathrm{d}\tilde{F}_{sdj,t}(p) = \frac{A_{sj}(c_{sdj,t})^{-\theta_j}}{\Phi_{dj,t}}, \tag{B.7}$$

where $\Phi_{dj,t} \equiv \sum_n A_{sj}(c_{sdj,t})^{-\theta_j}$.

For products in $\Omega_{j,t}^{0}$, the distribution of prices $G_{sdj,t}^{0}(p)$ paid in country d on products sourced from country s equals the overall distribution of prices paid in country d: $G_{dj,t}^{0}(p)$. For any given source country s:

$$G_{sdj,t}^{0}(p) = \mathbb{P}\left[p_{dj,t}(\omega) \leq p \middle| s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] = 1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}.$$

The unconditional distribution is the same as the distribution conditional on each source country, so

$$\begin{aligned} G_{dj,t}^{0}(p) &= \sum_s \mathbb{P}\left[p_{dj,t}(\omega) \leq p \middle| s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] \mathbb{P}\left[s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] \\ &= \sum_s \left(1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}\right) \lambda_{sdj,t}^{0} = 1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}, \end{aligned} \tag{B.8}$$

where the last equality follows from the fact that $\sum_s \lambda_{sdj,t}^{0} = 1$.

Putting these results together, we can now solve for the expenditure share within partition 0. Starting from the definition of expenditure shares,

$$\begin{aligned}
\lambda^0_{sdj,t} &\equiv \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \left(p_{sdj,t}(\omega)\right)^{1-\sigma_j} \mathrm{d}\omega}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \left(p_{ndj,t}(\omega)\right)^{1-\sigma_j} \mathrm{d}\omega} \\
&= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{sdj,t}\mathrm{d}\omega}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{ndj,t}\mathrm{d}\omega} \\
&= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t}}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t}} \\
&= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega}{\int_{[0,1]} \mathbf{1}\left\{\omega \in \Omega^0_{j,t}\right\} \mathrm{d}\omega} \\
&= \frac{\mu_{j,t}(0)\mathbb{P}\left[s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right]}{\mu_{j,t}(0)} \\
&= \frac{A_{sj}(c_{sdj,t})^{-\theta_j}}{\Phi_{dj,t}},
\end{aligned} \tag{B.9}$$

where $\mu_{i,t}(0)$ is the measure of the set $\Omega^0_{i,t}$. The third line uses the fact again that the distribution of prices conditional on the source country is the same as the unconditional distribution of prices, and the last equality uses the probability that a given source country hosts the lowest-cost supplier.

We can derive the corresponding ideal price indices using

$$\begin{aligned}
\left(P^0_{dj,t}\right)^{1-\sigma_j} &= \int_{\Omega^0_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j} \mathrm{d}\bar{\omega} = \int_{\Omega^*_{j,t}0} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t}\,\mathrm{d}\bar{\omega} \\
&= \int_{\Omega^0_{j,t}} \int_0^\infty (p)^{1-\sigma_j}\, \theta_j \Phi_{dj,t} p^{\theta_j - 1} \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\} \mathrm{d}p\,\mathrm{d}\bar{\omega}.
\end{aligned}$$

For a change of variables, define $x \equiv p_j^\theta \Phi_{dj,t}$, which implies that $\mathrm{d}x = \theta_j \Phi_{dj,t} p^{\theta_j - 1}\mathrm{d}p$ and $p = \left(x/\Phi_{dj,t}\right)^{1/\theta_j}$. Denoting $\gamma_j \equiv \Gamma\left([\theta_j + 1 - \sigma_j]/\theta_j\right)$, we can then rewrite the integral above as

$$\left(P^0_{dj,t}\right)^{1-\sigma_j} = \int_{\Omega^0_{j,t}} \int_0^\infty \left(\frac{x}{\Phi_{dj,t}}\right)^{\frac{1-\sigma_j}{\theta_j}} \exp\{-x\}\,\mathrm{d}x\,\mathrm{d}\bar{\omega} = \gamma_j\, \mu_{j,t}(0) \left(\Phi_{dj,t}\right)^{-\frac{1-\sigma_j}{\theta_j}}, \tag{B.10}$$

$\mu_{j,t}(0)$ denotes the measure of the set $\Omega^0_{j,t}$. The results show that, when firms are adjusting, trade shares operate as in the frictionless economy of EK.

Using standard hat algebra for changes in the common unit cost component $\hat{c}_{sdj,t} \equiv c_{sdj,t}/c_{sdj,t-1}$, we can express trade shares and price levels within partition $k = 0$ as:

$$\lambda^0_{sdj,t} = \frac{\lambda^0_{sdj,t-1}\hat{c}^{-\theta^j}_{sdj,t}}{\sum_n \lambda^0_{ndj,t-1}(\hat{c}_{ndj,t})^{-\theta^j}} \tag{B.11}$$

$$P^0_{dj,t} = P^0_{dj,t-1}\left[\sum_s \lambda^0_{sdj,t-1}(\hat{c}_{sdj,t})^{-\theta^j}\right]^{-\frac{1}{\theta^j}}. \tag{B.12}$$

We next derive an analogous result for partitions $k > 0$ when firms are not adjusting their extensive margin of suppliers.

B.3 Trade Shares When Firms Are Not Adjusting ($k > 0$)

For intermediate goods $\omega \in \Omega^k_{j,t}$, assemblers last adjusted the least-cost supplier $t - k$ periods ago. In order to account for changes in trade shares and price levels, we therefore need to recall optimal sourcing choices at period $t - k$ and trace changes in parameters and prices since $t - k$.

Suppose that in period $t - k$ intermediate good ω was optimally sourced from country s to country d in industry j. Then the destination price in period t for this intermediate good will be:

$$p_{sdj,t}(\omega) = \frac{c_{sdj,t}}{z_{sj}(\omega)} = \frac{\prod_{\varsigma=t-k+1}^{t} c_{sdj,t-k}\,\hat{c}_{sdj,\varsigma}}{z_{sj}(\omega)} = p_{sdj,t-k}(\omega)\prod_{\varsigma=t-k+1}^{t}\left(\hat{c}_{sdj,\varsigma}\right), \tag{B.13}$$

which is the initial destination price adjusted for the cumulative changes in trade costs and factor costs. Using this result, we can derive country d's expenditure share by source country across intermediate goods $\omega \in \Omega^k_{j,t}$

$$\begin{aligned}
\lambda^k_{sdj,t} &\equiv \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \left(p_{sdj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \left(p_{ndj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{sdj,t-k} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{ndj,t-k} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{dj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{dj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\mu_{j,t}(k)\lambda_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \mu_{j,t}(k)\lambda_{ndj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\lambda^0_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \lambda^0_{ndj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}},
\end{aligned} \tag{B.14}$$

where $\mu_{i,t}(k)$ is the measure of the set $\Omega^k_{i,t}$. The third line again uses the fact that, at $t-k$, the distribution of prices conditional on the source is the same as the unconditional distribution; and the last line uses the result from the previous section that $\lambda^0_{sdj,t-k} = \mathbb{P}\left[s = \arg\min_s \left\{p_{sdj,t-k}(\omega)\right\}\right]$.

We can derive the corresponding ideal price indices using

$$\begin{aligned}
\left(P^k_{dj,t}\right)^{1-\sigma_j} &= \int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j} \mathrm{d}\bar{\omega} \\
&= \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \left(p_{sdj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega \\
&= \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{sdj,t-k} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \\
&= \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t-k} \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \\
&= \frac{\mu_{j,t}(k)}{\mu_{j,t-k}(0)} \left(P^0_{dj,t-k}\right)^{1-\sigma_j} \sum_s \lambda^0_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \qquad \text{(B.15)}
\end{aligned}$$

The price level change in partition 0 satisfies $P^0_{dj,t} = P^0_{dj,t-1} \left[\sum_s \lambda^0_{sdj,t-1} (\hat{c}_{sdj,t})^{-\theta^j}\right]^{-\frac{1}{\theta^j}}$ by (B.10), so we can rewrite the ideal price for composite goods with the last supplier selection k periods ago

$$\left(P^k_{dj,t}\right)^{1-\sigma_j} = \frac{\mu_{j,t}(k)}{\mu_{j,t-k}(0)} \left(P^0_{dj,t-k-1}\right)^{1-\sigma_j} \left[\sum_n \lambda^0_{ndj,t-k-1} \hat{c}^{-\theta^j}_{ndj,t-k}\right]^{-\frac{1-\sigma_j}{\theta^j}} \sum_s \lambda^0_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}.$$

Denoting $\gamma_j \equiv \Gamma\left([\theta_j + 1 - \sigma_j]/\theta_j\right)$ and using the fact that $\left(P^0_{dj,t}\right)^{1-\sigma_j} = \mu_{j,t}(0) \left(\Phi_{dj,t}\right)^{-\frac{1-\sigma_j}{\theta_j}} \gamma_j$, we can rewrite the expression above as:

$$\left(P^k_{dj,t}\right)^{1-\sigma_j} = \gamma_j \mu_{j,t}(k) \left(\Phi_{dj,t-k}\right)^{-\frac{1-\sigma_j}{\theta_j}} \sum_s \left[\lambda^0_{sdj,t-k-1} A_{sj} \hat{c}^{-\theta^j}_{sdj,t-k}\right]^{-\frac{1-\sigma_j}{\theta^j}} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \qquad \text{(B.16)}$$

after expressing $\lambda^0_{sdj,t-k}$ recursively.

B.4 Aggregation Over Partitions

The aggregate ideal price level of the final good can be rewritten as a combination of the price levels of the partial price indices for the composites of intermediate goods purchased at time t from suppliers chosen $t-k$ periods ago:

$$\left(P_{dj,t}\right)^{1-\sigma_j} = \int_{[0,1]} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega} = \sum_{k=0}^{\infty}\int_{\Omega_{j,t}^k} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega} = \sum_{k=0}^{\infty}\left(P_{dj,t}^k\right)^{1-\sigma_j}.$$

Using the price index expressions (B.10) and (B.16) from the preceding subsections yields

$$\begin{aligned}\left(P_{dj,t}\right)^{1-\sigma_j} &= \gamma_j \sum_{k=0}^{\infty}\mu_{j,t}(k)\left(\Phi_{dj,t-k}\right)^{-\frac{1-\sigma_j}{\theta_j}}\sum_s\left[\lambda^0_{sdj,t-k-1}A_{sj}\,\hat{c}_{sdj,t-k}^{-\theta^j}\right]^{-\frac{1-\sigma_j}{\theta^j}} \\ &\qquad\times\exp\left\{\mathbf{1}\{k>0\}\log\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}\right\} \\ &= \sum_{k=0}^{\infty}\frac{\mu_{j,t}(k)}{\mu_{j,t-k}(0)}\left(P^0_{dj,t-k-1}\right)^{1-\sigma_j}\sum_n\left[\lambda^0_{ndj,t-k-1}\hat{c}_{ndj,t-k}^{-\theta^j}\right]^{-\frac{1-\sigma_j}{\theta^j}} \\ &\qquad\times\exp\left\{\mathbf{1}\{k>0\}\log\left[\sum_s\lambda^0_{sdj,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}\right]\right\}. \end{aligned} \tag{B.17}$$

Recall that, by optimal demand, expenditure shares of each partition relative to total expenditures are

$$\frac{P^k_{dj,t}Y^k_{dj,t}}{P_{dj,t}Y_{dj,t}} = \left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j}$$

Total expenditure shares are therefore simply the weighted average of trade shares across partitions

$$\lambda_{sdj,t} \equiv \sum_{k=0}^{\infty}\frac{P^k_{dj,t}Y^k_{dj,t}}{P_{dj,t}Y_{dj,t}}\lambda^k_{sdj,t} = \sum_{k=0}^{\infty}\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j}\lambda^k_{sdj,t}, \tag{B.18}$$

which can also be stated as

$$\lambda_{sdj,t} = \left(\frac{P^0_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j}\frac{\lambda^0_{sdj,t-1}\hat{c}_{sdj,t}^{-\theta^j}}{\sum_n\lambda^0_{ndj,t-1}\hat{c}_{ndj,t}^{-\theta^j}} + \sum_{k=1}^{\infty}\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j}\frac{\lambda^0_{sdj,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n\lambda^0_{ndj,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}}.$$

Writing $\lambda^0_{sdj,t-k}$ and $\lambda^0_{ndj,t-k}$ recursively, we can express trade shares compactly as

$$\lambda_{sdj,t} = \sum_{k=0}^{\infty} \left(\frac{P^k_{dj,t}}{P_{dj,t}} \right)^{1-\sigma_j} \frac{\lambda^0_{sdj,t-k-1} \hat{c}^{-\theta^j}_{sdj,t-k} \exp\left\{ \mathbf{1}\{k > 0\} \log \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma} \right)^{1-\sigma_j} \right\}}{\sum_n \lambda^0_{ndj,t-k-1} \hat{c}^{-\theta^j}_{ndj,t-k} \exp\left\{ \mathbf{1}\{k > 0\} \log \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma} \right)^{1-\sigma_j} \right\}}. \quad \text{(B.19)}$$

B.5 Convergence

Results in the preceding subsection imply that trade shares can be expressed a sum over infinitely many partitions. We now establish regularity conditions for convergence.

Lemma 1 (Convergence)**.** *If cumulative changes in trade costs are finite-valued* $\lim_{k\to\infty} |\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$*, then price levels* $P^k_{dj,t} < \infty$ *and trade shares* $0 < \lambda_{dj,t} < 1$ *are finite-valued.*

Proof. Note that $\left(\Phi_{dj,t-k}\right)^{(\sigma_j-1)/\theta_j} < \infty$ and $\sum_s \left[\lambda^0_{sdj,t-k-1} A_{sj} \hat{c}^{-\theta^j}_{sdj,t-k} \right]^{(\sigma_j-1)/\theta_j} < \infty$ are both finite-valued, because they are equilibrium objects of a static equilibrium of the model. Also note that, for any $k > m$, if $|\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, then $|\prod_{\varsigma=t-m+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, since the product up to k includes every term in the product up to m. Therefore, if $\lim_{k\to\infty} |\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, then, for every $k < \infty$, the product will also be finite. It follows that $P^k_{dj,t} < \infty$ is finite valued for every k. Given that $\lim_{k\to\infty} \mu_{j,t}(k) = \lim_{k\to\infty} (1-\zeta_j)^k \zeta_j = 0$. These findings also guarantee that $P_{dj,t} < \infty$. □

B.6 Proofs

B.6.1 Proof of Proposition 1.

When the economy is in steady state, then for any $t <$ changes must satisfy $\hat{\mathbf{F}}_t = \hat{\mathbf{F}}_{\not{k}}$ and $\hat{\mathbf{w}}_t = \hat{\mathbf{w}}_{\not{k}}$ so that $\hat{c}_{s,t} = 1$ for all $s \in \mathcal{D}$. For the firms that are adjusting at t ($k = 0$), evaluating Equation (19) at those values, $\lambda^0_{sdj,t} = \lambda^0_{sdj,t-1} = \cdots = \lambda^0_{sdj,0}$ for all t. For the firms that are not adjusting at t ($k > 0$), we have $t - k > 0$ in equilibrium as long as the partition exists and can evaluate Equation (19) using the same logic as above: $\lambda^k_{sdj,t} = \lambda^0_{sdj,t-k} = \lambda^0_{sdj,0}$ for all t. From Equation (19), it is easy to see that $\lambda_{sdj,t} = \lambda^0_{sdj,t}$, which shows that $\boldsymbol{\lambda}_t = \boldsymbol{\lambda}^{EK}$ in steady state.

To derive the stationary distribution of contract lengths, begin by noting that the case $k = 0$ is trivial, since $\mu(0) = \mathbb{P}[K_t = 0] = \zeta_j$ does not vary. Now consider the case $k > 0$. Note that:

$$
\begin{aligned}
\mathbb{P}[K_t = k, k > 0] &= \sum_{l=0}^{\infty} \mathbb{P}[K_t = k, k > 0 | K_{t-1} = l]\, \mathbb{P}[K_{t-1} = l] \\
&= (1 - \zeta_j)\mathbb{P}[K_{t-1} = k - 1]
\end{aligned}
$$

The remaining proof for $k > 0$ then follows by induction. For $K_t = 1$, $\mathbb{P}[K_t = 1] = (1 - \zeta_j)\zeta_j$, and for $K_t = 2$, $\mathbb{P}[K_t = 2] = (1 - \zeta_j)\mathbb{P}[K_{t-1} = 1] = (1 - \zeta_j)^2\zeta_j$, and so forth recursively, for an arbitrary $K_t = k$ we must have $\mathbb{P}[K_t = k] = (1 - \zeta_j)^k\zeta_j$. This is the probability density function of a geometric distribution with mean $(1 - \zeta_j)/\zeta_j$ and standard deviation $\sqrt{1 - \zeta_j}/\zeta_j$.

Finally, using the definition of the measure μ, $\mu_{j,t}(k) = \mathbb{P}[K_t = k]$ for $t \geq k$. Given the Markov property of K_t, the following distribution will be stationary for all $k \in \mathbb{N}_0$:

B.6.2 Proof of Proposition 2.

For ease of notation, we suppress sector subscripts throughout the derivations. Consider a one-time permanent change in trade costs such that $\hat{\tau}_{sd,t} \neq 1$ and $\hat{\tau}_{sd,t+h} = 1\ \forall h > 0$. To characterize the partial trade elasticity at horizon h, we first characterize the elasticity for trade shares of each partition, then aggregate them up using the consumption shares derived from the CES preferences over partitions. The change in expenditure shares on intermediate goods in the kth partition in period $t+h$, relative to period $t-1$ is given by

$$
\log \frac{\lambda^k_{sd,t+h}}{\lambda^k_{sd,t-1}} = \begin{cases}
-(\sigma - 1)\log \hat{\tau}_{sd,t} + \log \frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} \left(\frac{(c_{s,t+h}/P^k_{d,t+h})}{(c_{s,t+h-k}/P^k_{d,t+h-k})} \right)^{1-\sigma} & , k \geq h \\
\log \frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} \left(\frac{(c_{s,t+h}/P^k_{d,t+h})}{(c_{s,t+h-k}/P^k_{d,t+h-k})} \right)^{1-\sigma} & , 1 \leq k < h \\
\log \frac{\lambda^0_{sd,t+h-1}}{\lambda^k_{sd,t-1}} \left(\frac{(c_{s,t+h}/P^0_{d,t+h})}{(c_{s,t-1}/P^0_{d,t-1})} \right)^{\theta} & , k = 0
\end{cases}
$$

The first line denotes intermediate goods that have not updated suppliers since the shock arrived. For such intermediate goods, changes in expenditure shares still explicitly depend on the shock to trade costs. The remaining intermediate goods have updated at least once, and a "new" optimal sourcing share $\lambda^0_{sd,t+h-k}$ from a time period between t and $t + h$ encodes the "initial price index" relative to which changes in expenditure shares are updated as well as the effect of the shock in trade costs. Unit costs are the relevant GE variables.

Denote

$$\Delta \boldsymbol{G}^{EK}_{sd,t,t+h} = -\theta \log \prod_{k=1}^{h} \frac{\hat{c}_{sd,t+k}}{\hat{P}^0_{sd,t+k}}$$

and

$$\Delta \boldsymbol{G}^{k}_{sd,\varsigma,t+h} = (1-\sigma) \log \prod_{\varsigma'=\varsigma+1}^{t+h} \frac{\hat{c}_{sd,\varsigma'}}{\hat{P}^k_{sd,\varsigma'}}$$

Then we can solve backwards to express all changes in trade shares above in terms of $\lambda^0_{sd,t-1}$, if possible:

$$\log \frac{\lambda^k_{sd,t+h}}{\lambda^k_{sd,t-1}} = \begin{cases} -(\sigma-1)\log\hat{\tau}_{sd,t} + \log\frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} + \Delta \boldsymbol{G}^k_{sd,t,t+h} & , k \geq h \\ -\theta\log\hat{\tau}_{sd,t} + \log\frac{\lambda^0_{sd,t-1}}{\lambda^k_{sd,t-1}} + \Delta \boldsymbol{G}^{EK}_{sd,t,t+h-k} + \Delta \boldsymbol{G}^k_{sd,t+h-k,t+h} & , 1 \leq k < h \\ -\theta\log\hat{\tau}_{sd,t} + \Delta \boldsymbol{G}^{EK}_{sd,t,t+h} & , k = 0 \end{cases}$$

Use the fact that outcomes determined at t and earlier do not respond to the change in trade costs. Hence, the elasticity of $\lambda^k_{sd,t+h}$ with respect to a change in trade costs at t, is hence given by,

$$\frac{\mathrm{d}\log(\lambda^k_{sd,t+h}/\lambda^k_{sd,t})}{\mathrm{d}\log\tau_{sd,t}} = \begin{cases} -(\sigma-1) + \frac{\mathrm{d}\Delta \boldsymbol{G}^k_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} & , k \geq h \\ -\theta + \frac{\mathrm{d}\Delta \boldsymbol{G}^{EK}_{sd,t,t+h-k}}{\mathrm{d}\log\tau_{sd,t}} + \frac{\mathrm{d}\Delta \boldsymbol{G}^k_{sd,t+h-k,t+h}}{\mathrm{d}\log\tau_{sd,t}} & , 1 \leq k < h \\ -\theta + \frac{\mathrm{d}\Delta \boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} & , k = 0 \end{cases}$$

To a first order, the change in overall expenditures at time $t+h$ caused by a one-time permanent shock to trade costs at t is given by

$$\begin{aligned}
\frac{\mathrm{d}\log(\lambda_{sd,t+h}/\lambda_{sd,t})}{\mathrm{d}\log\tau_{sd,t}} =& \sum_{k=0}^{\infty}\omega_k\left\{\frac{\mathrm{d}\log\lambda^k_{sd,t+h}/\lambda^k_{sd,t}}{\mathrm{d}\log\tau_{sd,t}} + (1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
=& \sum_{k=0}^{h-1}\omega_k\left\{-\theta + \frac{\mathrm{d}\Delta\boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} + \frac{\mathrm{d}\Delta\boldsymbol{G}^{k}_{sd,t+h-k,t+h}}{\mathrm{d}\log\tau_{sd,t}} + (1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
&+ \sum_{k=h}^{\infty}\omega_k\left\{(1-\sigma) + \frac{\mathrm{d}\Delta\boldsymbol{G}^{k}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} + (1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
=& -\theta\sum_{k=0}^{h-1}\omega_k + (1-\sigma)\sum_{k=h}^{\infty}\omega_k \\
&+ \sum_{k=0}^{h-1}\omega_k\frac{\mathrm{d}\Delta\boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} + \sum_{k=0}^{h-1}\omega_k(1-\sigma)\left\{\frac{\sum_{\varsigma=t+h-k+1}^{t+h}\mathrm{d}\log c_{sd,\varsigma}}{\mathrm{d}\log\tau_{sd,t}} + \frac{\sum_{\varsigma=t}^{t+h-k}\mathrm{d}\log P^k_{sd,\varsigma}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
&+ \sum_{k=h}^{\infty}\omega_k(1-\sigma)\left\{\frac{\sum_{i=0}^{t+h}\mathrm{d}\log c_{sd,t+i}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
&- (1-\sigma)\frac{\sum_{i=0}^{h}\mathrm{d}\log P_{sd,t+i}}{\mathrm{d}\log\tau_{sd,t}}
\end{aligned}$$

where $\omega_k \equiv \frac{\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma}\lambda^k_{sdj,t}}{\sum_k\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma}\lambda^k_{sdj,t}} = \frac{\mu_t(k)\lambda^k_{sdj,t}}{\sum_k\mu_t(k)\lambda^k_{sdj,t}}$. If t was a steady state, then $\omega_k = \mu(k)$, and the partial horizon-h trade elasticity equals:

$$\varepsilon^{t+h}_{sd} \equiv \frac{\partial\log\lambda_{sdj,t+h}}{\partial\log\tau_{sd,t}} = -\theta\sum_{k=0}^{h-1}\mu(k) + (1-\sigma)\sum_{k=h}^{\infty}\mu(k).$$

Using the stationary distribution of $\mu_t(k)$ to substitute for $\mu(k)$, we obtain the expression stated in the main text.

B.7 Proof of Proposition 3.

We begin by rearranging Equation (19) to express the prices of composite goods in terms of home expenditure shares:

$$\begin{aligned}
\lambda_{ddi,t} P_{d,t}^{1-\sigma_i} &= \mu_i(0)\gamma_i \left(\Phi^0_{ddi,t}\right)^{-\left(\frac{1-\sigma_i}{\theta_i}\right)} \frac{c_{dd,i}^{-\theta_i}}{\Phi_{ddi,t}} + \textstyle\sum_{k\geq 1} \gamma_i \mu_i(k) \left(\Phi^0_{ddi,t-k}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda^0_{ddi,t-k} \left(\frac{c_{ddi,t}}{c_{ddi,t-k}}\right)^{1-\sigma_i} \\
&= \mu_i(0)\gamma_i \left(\frac{c_{dd}^{-\theta_i}}{\lambda^{k=0}_{dd_i t}}\right)^{-\left(\frac{1-\sigma_i}{\theta_i}\right)} \lambda^{k=0}_{ddi,t-k} + \textstyle\sum_{k\geq 1} \gamma_i \mu_i(k) \left(\frac{c_{dd,t-k}^{-\theta_i}}{\lambda_{ddi,t-k}}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda^0_{ddi,t-k} \left(\frac{c_{dd,t}}{c_{dd,t-k}}\right)^{1-\sigma_i} \\
P_{di,t}^{1-\sigma_i} &= c_{dd,t}^{1-\sigma_i} \left(\lambda^{k=0}_{ddi,t}\right)^{\frac{1-\sigma_i}{\theta_i}} \frac{1}{\lambda_{ddi,t}} \gamma_i \left[\mu_i(0)\lambda^{k=0}_{ddi,t-k} + \textstyle\sum_{k\geq 1} \mu_i(k) \left(\frac{\lambda^{k=0}_{ddi,t}}{\lambda^{k=0}_{ddi,t-k}}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda^0_{ddi,t-k}\right]
\end{aligned}$$

where the second line follows from using the trade shares across varieties with different adjustment status k to substitute for the terms $\Phi^k_{ddi,t}$. Substituting for the common component of unit cost, we then obtain:

$$\frac{P_{di,t}}{w_{d,t}} = \left(\lambda^0_{ddi,t}\right)^{\frac{1}{\theta_i}} \left(\lambda_{ddi,t}\right)^{1/(\sigma_j-1)} \left(\xi_{di,t}\right)^{1/(1-\sigma_i)} \prod_j \left(\frac{P_{dk,t}}{w_{s,t}}\right)^{\alpha_{sji}},$$

where

$$\xi_{di,t} \equiv \mu_i(0)\lambda^0_{ddi,t} + \sum_{k\geq 1} \mu_i(k) \left(\frac{\lambda^0_{ddi,t-k}}{\lambda^0_{ddi,t}}\right)^{\frac{\sigma_i-1}{\theta_i}} \lambda^0_{ddi,t-k}.$$

Taking logs,

$$\log \frac{P_{di,t}}{w_{d,t}} = \log B_{si,t} + \sum_j \alpha_{sji} \log \frac{P_{sj,t}}{w_{s,t}}$$

where $B_{di,t} \equiv A_i^{-\frac{1}{\Theta_i}\alpha_{di}} \gamma_i \left(\lambda^0_{ddi,t}\right)^{\frac{1}{\theta_i}} \left(\lambda_{ddi,t}\right)^{1/(\sigma_j-1)} \left(\xi_{dj,t}\right)^{1/(1-\sigma_i)}$ In matrix notation, this leads to

$$(\mathbf{I} - A_s) \log \hat{\boldsymbol{P}}_{s,t} = \log \boldsymbol{B}_{s,t},$$

where $A_d = \left\{\alpha_{dji}\right\}$ and $\log \hat{\boldsymbol{P}}_{d,t}$ and $\log \boldsymbol{B}_{d,t}$ are $I \times 1$ vectors. Inverting this system of equations, we obtain

$$\frac{P_{di,t}}{w_{d,t}} = \prod_j B_{dj,t}^{\bar{a}_{dji}},$$

where $\bar{a}_{dji}$ is the (j, i) entry of the Leontief matrix $(\mathbf{I} - A_d)^{-1}$. This implies

$$P_{di,t} = \gamma_i^{1/(1-\sigma_i)} w_{di,t} \prod_{j,j} \left[A_j^{-\frac{\bar{a}_{sji}\alpha_{sji}}{\Theta_j}} \left(\lambda^{k=0}_{ddj,t}\right)^{\frac{\bar{a}_{sji}}{\theta_j}} \left(\lambda_{ddi,t}\right)^{\bar{a}_{dji}/(\sigma_j-1)} \left(\xi_{dj,t}\right)^{\frac{\bar{a}_{sji}}{1-\sigma_j}}\right]$$

Hence, the consumer price index in country d is given by:

$$P_{d,t} = \prod_i \left(P_{di,t}\right)^{\eta_i} = \Theta_s w_{d,t} \prod_{i,j} \left[A_j^{-\alpha_{sji}/\theta_j} \left(\lambda_{ddj,t}^{k=0}\right)^{\frac{1}{\theta_j}} \left(\lambda_{ddj,t}\right)^{\frac{1}{\sigma_j - 1}} \left(\xi_{dj,t}\right)^{\frac{1}{1-\sigma_j}} \right]^{\eta_i \bar{a}_{sji}},$$

where Θ_s is a collection of time-invariant parameters. Consequently, the real wage is given by:

$$W_{d,t} \equiv \frac{w_{d,t}}{P_{d,t}} = \Theta_s^{-1} \prod_{i,j} \left[A_j^{\alpha_{sji}/\theta_j} \left(\lambda_{ddj,t}^{k=0}\right)^{-\frac{1}{\theta_j}} \left(\lambda_{ddj,t}\right)^{-\frac{1}{\sigma_j - 1}} \left(\xi_{dj,t}\right)^{\frac{1}{\sigma_j-1}} \right]^{\eta_i \bar{a}_{sji}},$$

while the change in real wages between $t-1$ and $t+h$ equals:

$$\frac{W_{d,t+h}}{W_{d,t}} = \prod_{i,j} \left[\left(\frac{\lambda_{ddj,t+h}^{k=0}}{\lambda_{ddj,t-1}^{k=0}}\right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}}\right)^{-\frac{1}{\sigma_j - 1}} \left(\frac{\xi_{dj,t+h}}{\xi_{dj,t-1}}\right)^{\frac{1}{\sigma_j-1}} \right]^{\eta_i \bar{a}_{sji}},$$

If $t-1$ is a steady state, then $\lambda_{ddj,t-1}^{k} = \lambda_{ddj,t-1}$ for all $k \in \{0, 1, 2, ...\}$ and the above expression simplifies to:

$$\begin{aligned} \frac{W_{d,t+h}}{W_{d,t}} &= \prod_{i,j} \left[\left(\frac{\lambda_{ddj,t+h}^{k=0}}{\lambda_{ddj,t-1}}\right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda_{ddj,t+h}}{\xi_{dj,t+h}}\right)^{-\frac{1}{\sigma_j - 1}} \right]^{\eta_i \bar{a}_{sji}} \\ &= \prod_{i,j} \left[\left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}}\right)^{-\frac{1}{\theta_j}} \left(\Xi_{dj,t}\right)^{\frac{1}{\sigma_j - 1}} \right]^{\eta_i \bar{a}_{sji}}, \end{aligned}$$

where:

$$\Xi_{dj,t} \equiv \zeta_j \left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t+h}^{0}}\right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}} + \sum_{k=1}^{h} \zeta_j (1-\zeta_j)^k \left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t+h-k}^{0}}\right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}} + \zeta_j (1-\zeta_j)^{h+1} \left(\frac{\lambda_{ddi,t+h}}{\lambda_{ddj,t-1}}\right)^{\frac{\sigma_j - 1 - \theta_j}{\theta_j}}.$$